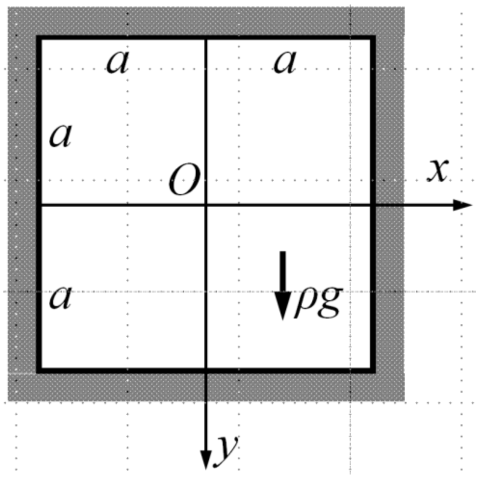
1. Consider a square thin-plate that is clamped at all edges and subjected to a uniform body force field $F_x = 0, F_y = \rho g$. Using Ritz method determine the displacements and stresses for a trial displacement field

$$u = A_1\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{a^2}\right)\frac{x}{a}\frac{y}{a}, \quad v = B_1\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{a^2}\right).$$

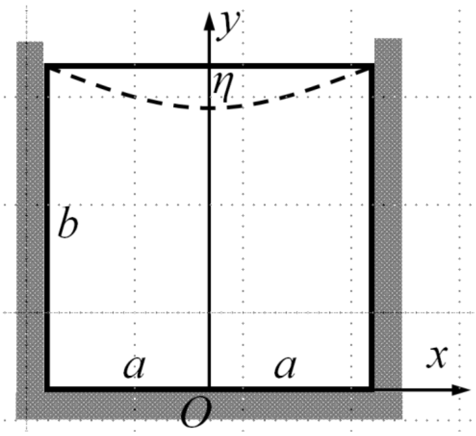
To simplify the calculation, assume zero Poisson ratio ($\nu = 0$).



2. Consider a thin-plate that is clamped at three edges and subjected to displacement boundary conditions $u = 0;\ v = -\eta\left(1 - x^2/a^2\right)$ along its top edge. Using Galerkin method determine the displacements and stresses for a trial displacement field

$$u = A_1\left(1-\frac{x^2}{a^2}\right)\frac{x}{a}\frac{y}{b}\left(1-\frac{y}{b}\right), \quad v = -\eta\left(1-\frac{x^2}{a^2}\right)\frac{y}{b} + B_1\left(1-\frac{x^2}{a^2}\right)\frac{y}{b}\left(1-\frac{y}{b}\right).$$

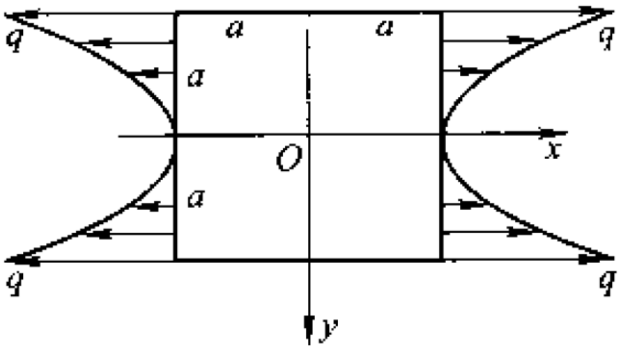
Assume zero body forces ($F_x = 0, F_y = 0$).



3. Consider a square thin-plate that is only subjected to traction boundary conditions $\sigma_x|_{x=\pm a} = q(y/a)^2$. Given the trial Airy stress function

$$\psi = \frac{qy^4}{12a^2} + qa^2\left(1-\frac{x^2}{a^2}\right)^2\left(1-\frac{y^2}{a^2}\right)^2\left(A_1 + A_2\frac{x^2}{a^2} + A_3\frac{y^2}{a^2} + \cdots\right),$$

use the principle of stress variation to determine the stress field. Consider two cases: including (a) A_1 only and (b) A_1, A_2, A_3. Assume zero body forces ($F_x = 0, F_y = 0$).



4. Given the trial Prandtl stress function $\psi = Axy\left(a^2 - x^2 - y^2\right)$, solve the torsion problem of a quarter of circular section by using the principle of stress variation.