1. For the Cauchy stress tensor with components

$$\begin{bmatrix} 10 & 25 & 0 \\ 25 & 20 & 0 \\ 0 & 0 & 30 \end{bmatrix} \text{MPa}$$

compute

(1) The traction vector acting on an internal material plane with normal $\mathbf{n} = (\mathbf{e}_1 - \mathbf{e}_2)/\sqrt{2}$;

(2) The normal and tangential tractions acting on the plane given in part (1);

(3) The principal stresses and principal directions;

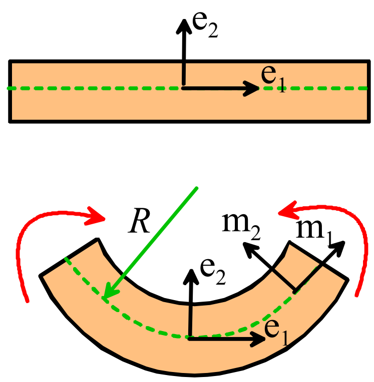
(4) The hydrostatic stress;

(5) The deviatoric stress tensor;

(6) The Von-Mises equivalent stress.

2. In this problem we consider further the beam bending calculation discussed in the previous homework (F03_Strain). Suppose that the beam is made from a material in which the Material Stress tensor (PK2) is related to the Lagrange strain tensor by

$$\Sigma_{ij} = 2GE_{ij}$$



(this can be regarded as representing an elastic material with zero Poisson's ratio and shear modulus G)

(1) Calculate the distribution of material stress (PK2) in the bar, expressing your answer as components in the $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ basis;

(2) Calculate the distribution of nominal stress (PK1) in the bar expressing your answer as components in the $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ basis;

(3) Calculate the distribution of Cauchy stress in the bar expressing your answer as components in the $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ basis;

(4) Repeat part (1) to part (3) but express the stresses as components in the $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$ basis;

(5) Calculate the distribution of traction on a surface in the beam that has normal $\mathbf{e}_1$ in the undeformed beam. Give expressions for the tractions in both $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$;

(6) Show that the surfaces of the beam that have positions $x_2 = \pm h/2$ in the undeformed beam are traction free after deformation;

(7) Calculate the resultant moment acting on the ends of the beam.

3. A solid is subjected to some loading that induces a Cauchy stress $\sigma_{ij}^{(0)}$ at some point in the solid. The solid and the loading frame are then rotated together so that the entire solid (as well as the loading frame) is subjected to a rigid rotation R_{ij}. This causes the components of the Cauchy stress tensor to change to new values $\sigma_{ij}^{(1)}$. The goal of this problem is to calculate a formula relating $\sigma_{ij}^{(0)}$, $\sigma_{ij}^{(1)}$ and R_{ij}.

(1) Let $n_i^{(0)}$ be a unit vector normal to an internal material plane in the solid before rotation. After rotation, this vector (which rotates with the solid) is $n_i^{(1)}$. Write down the formula relating $n_i^{(0)}$ and $n_i^{(1)}$;

(2) Let $T_i^{(0)}$ be the internal traction vector that acts on a material plane with normal $n_i^{(0)}$ in the solid before application of the rigid rotation. Let $T_i^{(1)}$ be the traction acting on the same material plane after rotation. Write down the formula relating $T_i^{(0)}$ and $T_i^{(1)}$;

(3) Using the definition of Cauchy stress, find the relationship between $\sigma_{ij}^{(0)}$, $\sigma_{ij}^{(1)}$ and R_{ij};

(4) Repeat the above calculation, but instead, calculate a relationship between the components of Nominal stress $S_{ij}^{(0)}$ and $S_{ij}^{(1)}$ before and after the rigid rotation;

(5) Repeat the above calculation, but instead, calculate a relationship between the components of material stress $\Sigma_{ij}^{(0)}$ and $\Sigma_{ij}^{(1)}$ before and after the rigid rotation.

4. The stress field

$$\sigma_{ij} = \frac{-3P_k x_k x_i x_j}{4\pi R^5} \qquad R = \sqrt{x_k x_k}$$

represents the stress state in an infinite, incompressible elastic solid ($\nu = 0.5$) that is subjected to a point force with components P_k acting at the origin (you can visualize a point force as a very large body force which is concentrated in a very small region around the origin).

(1) Verify that the stress field is in static equilibrium;

(2) Consider a spherical region of material centered at the origin. This region is subjected to (a) the point force acting at the origin; and (b) a force exerted by the stress field on the outer surface of the sphere. Calculate the resultant force exerted on the outer surface of the sphere by the stress, and show that it is equal in magnitude and opposite in direction to the point force.