Introduction

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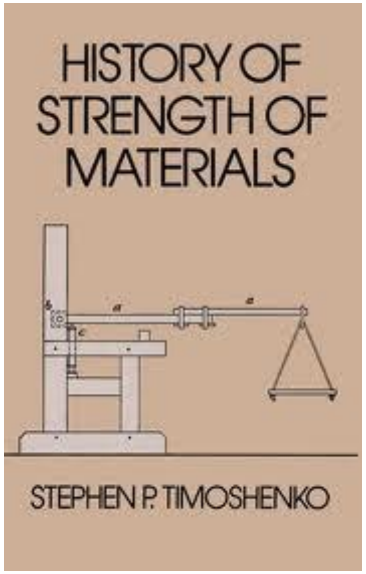
History of Strength of Materials

"From the earliest times when people started to build, it was found necessary to have information regarding the strength of structural materials so that rules for determining safe dimensions of members could be drawn up."

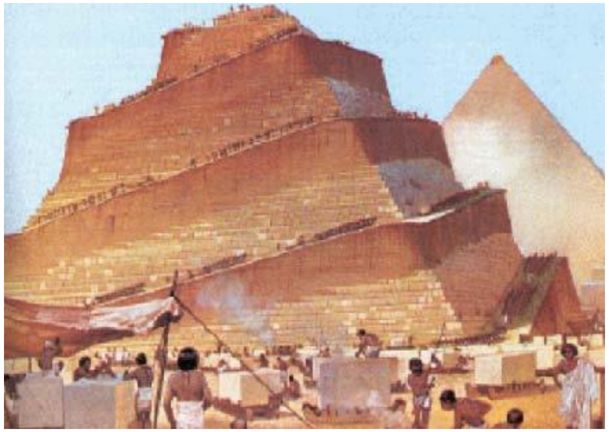


Old Kingdom of Egypt

Pyramids at Giza, Egypt ~ 2500 B. C.

Technology of Ancient Egypt

- Solar Calendar
- Accurate Surveying Techniques
- Mathematics and Spatial Concepts: Volumes (truncated pyramid), Slopes, Angles, Weight of Stones (Density), Multiplication & Division, Decimal System, $\pi \rightarrow 3.16$
- Simple Machines: Lever, Inclined plane, Leveling with water, No pulleys, No wheeled carts, Very little timber



Progression of Pyramid Design

Step Pyramid

- Built in stages, expanded later
- Inward leaning blocks
- Granite roof beam
- Below ground chambers

Bent Pyramid

- Poor foundation, subsidence
- Internal structural collapse
- Reduced angle by 10°
- Corbelled ceilings

Khufu's Pyramid

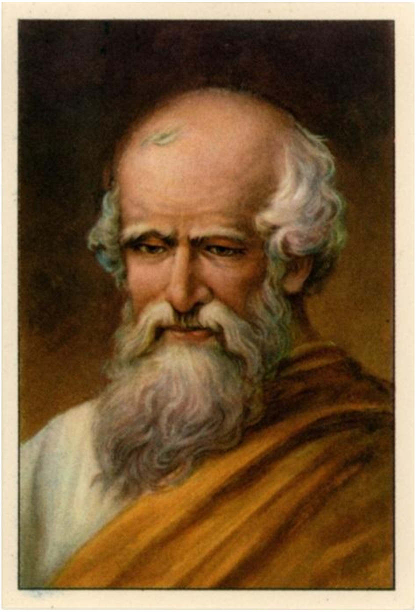
- 147 m in height, 51.9^0,
- Granite beams
- Facing stones fitted with a tolerance of less than 1 mm
- Stress relieving chambers

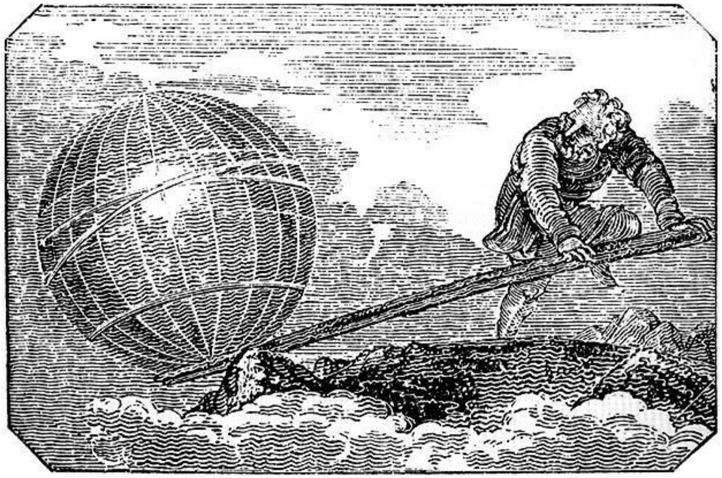
Lessons Learned by Trial and Error

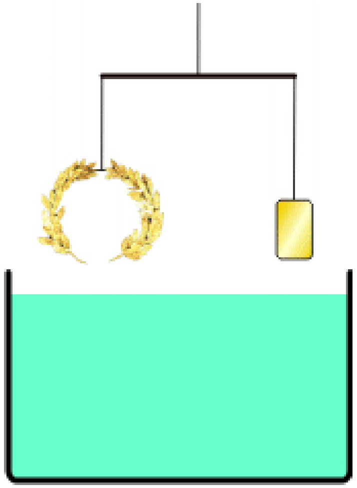
- Site must be solid rock (no shifting sand)
- Foundation must be strong enough to support the structure above it, but not be crushed or dislodged.
- Precise Measurements: to maintain shape and vertical plumb
- Masonry Courses: placed horizontally to distribute loads
- Strength of Materials Knowledge: Limestone vs. Granite
  - **Limestone**: readily available, soft – easy to work with, cut by hand, limited strength (< 3 m beams)
  - **Granite**: used for large beams in interior chambers because of its superior strength

Archimedes (287-212 BC)

- 唯一与现代科学相通的天才
- The Law of Lever: “Magnitudes are in equilibrium at distances reciprocally proportional to their weights.”
- Said to have discovered “Archimedes’ principle” while bathing and subsequently ran naked through the streets of Syracuse shouting “Eureka!”.



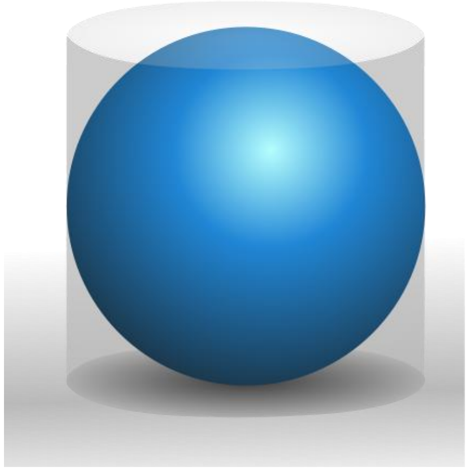




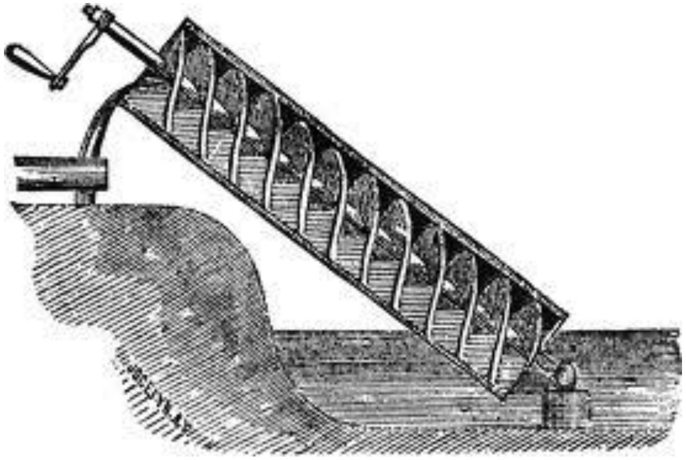
Archimedes (287-212 BC)

- Most known for geometric proofs and *may* have invented calculus.
- Invented the pulley, screw, and catapult; defined the law of levers.
- Outlined methods for finding the center of gravity of bodies.

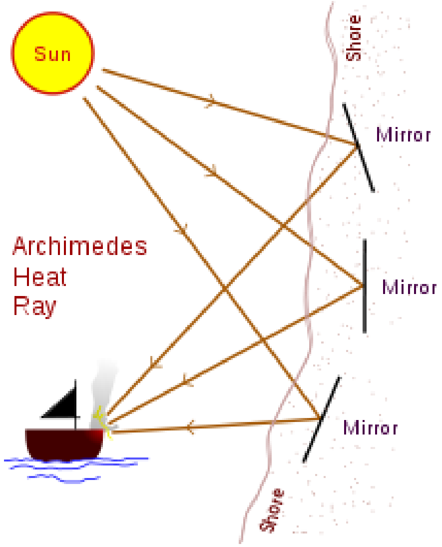
Archimedes portrait in Fields Medal



“On the Sphere & Cylinder”



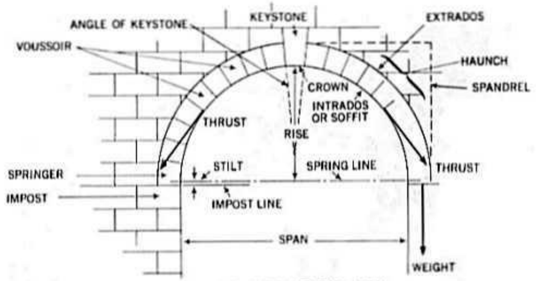
Archimedes’ Screw



Archimedes’ Heat Ray

Roman Empire

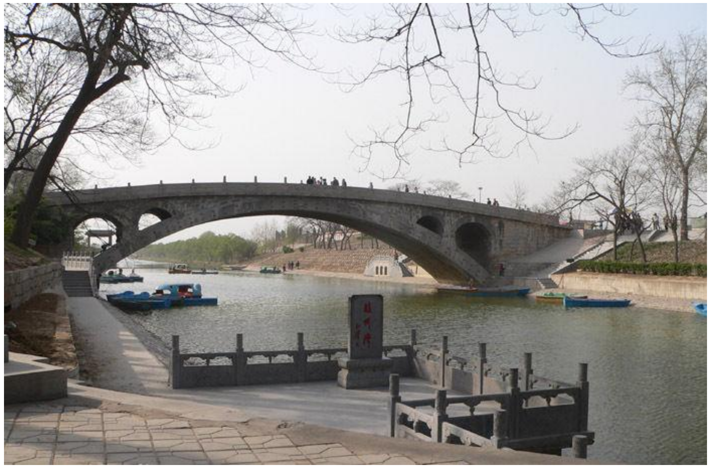
- Archimedes killed by Roman soldiers while doing research.
- The Romans did not understand strength of materials, so they built semi-circular arches of small span.
- *EI* is “Flexural Rigidity”: Romans used large *E* and *I*.
- Modern materials: when *E* is large *I* does not have to be large.



ELEMENTS OF THE ARCH

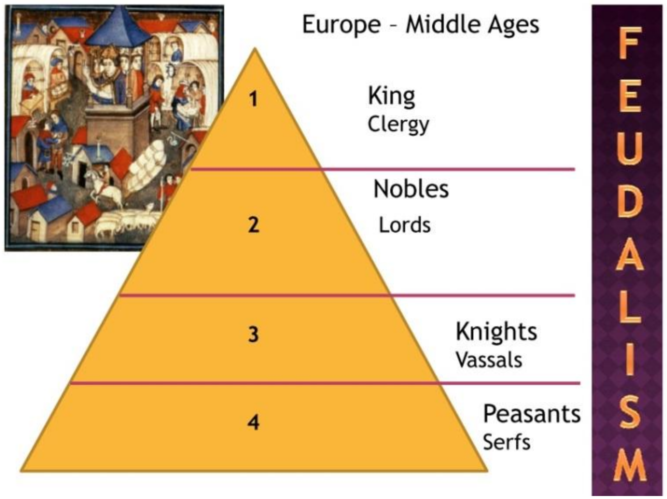
赵州桥， 600 A.D.， 隋朝

- 横跨河北省赵县洨河，由著名匠师李春设计和建造。
- Small circular arc (84°)
- Reduced building materials
- Water currents not disturbed by bridge supports
- Bridge abutments protected
- Preserving the water channel
- 45°span angle from abutments distributes load vertically and horizontally



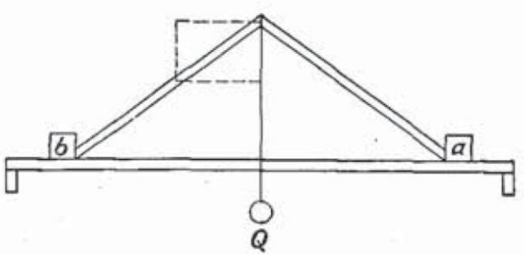
The Middle Ages

- “Most of the knowledge that Greeks and Romans accumulated in the way of structural engineering was lost during the Middle Ages and only since the Renaissance has it been recovered.”
- Fortunately, Arabians served as the knowledge keeper.



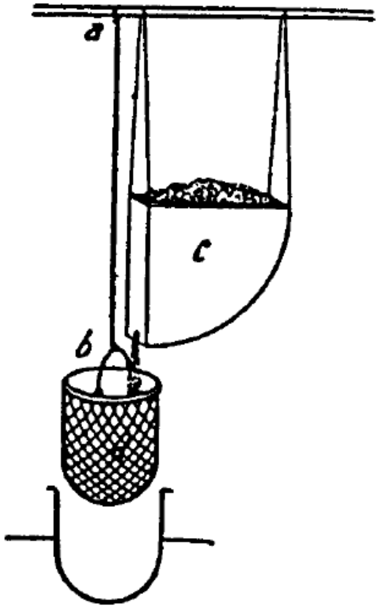
Leonardo da Vinci (1452-1519)

- Italian artist and scientist.
- A true Renaissance man who studied architecture, astronomy and anatomy.
- Conducted tensile tests on wires and related the measured displacement to the wire strength.
- Attempted to analyze columns , beams, and trusses.
- "Mechanics is the paradise of mathematical science because here we come to the fruits of mathematics."



Leonardo da Vinci (1452-1519)

- Load-carrying ability of columns: linearly proportional to diameter but inversely proportional to length.



- Load-carrying ability of beams:

➢ Varying inversely as length and directly as width for both end-supported and cantilever beams.

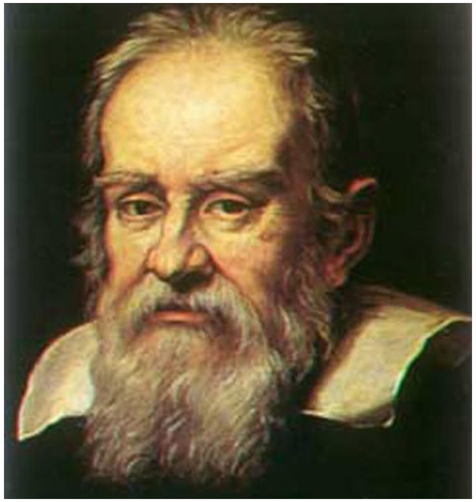
➢ The part that is farthest from the two supports will bend the most.

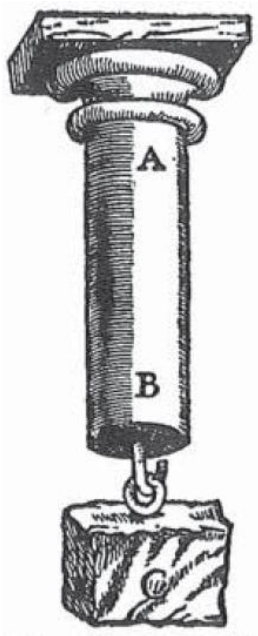
➢ The effect of beam depth was not discussed.

- These important results were unfortunately buried in da Vinci's notes. Engineers of the 15th and 16th centuries still relied on experience and judgment.

Galileo Galilei (1564-1642)

- Another true Renaissance man who is most known for his work in dynamics and astronomy.
- Performed the leaning Tower of Pisa experiment in (1589).
- “Two New Sciences” (1638) represents the beginning of the science of *strength of materials*.
- Studied the tensile strength of bars, and found that this strength (absolute resistance to fracture) is proportional to cross-sectional area, but independent of length.



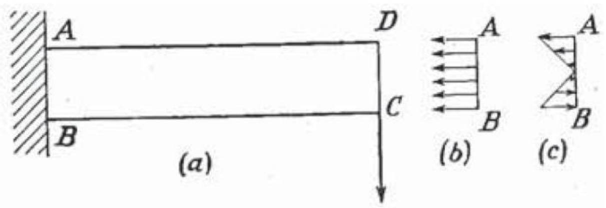


DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſsimo
Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

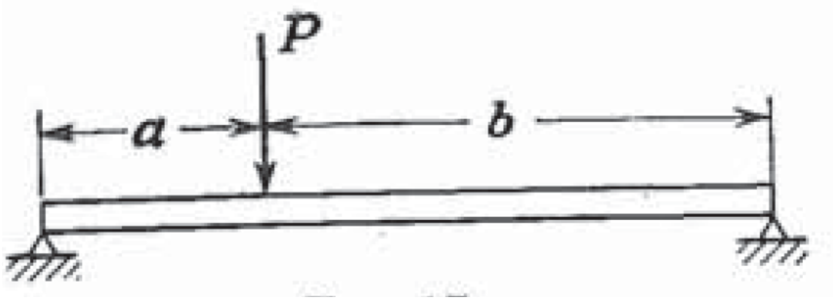
Galileo Galilei (1564-1642)

- Fracture occurs at the point *B*, serving as the fulcrum for the lever *BC*.
- The resistance (stress) is uniformly distributed along *AB*.
- Predicted a value three times larger than the correct resisting moment at the built-in. The same is true for the breaking load at *C*.
- For beams with $b>h$, the ratio $b:h$ holds between breaking loads along breadth and depth.
- The bending moment at the built-in, due to gravity, increases as the square of beam length.
- The resisting moment increases as the cube of the radius of a circular beam.

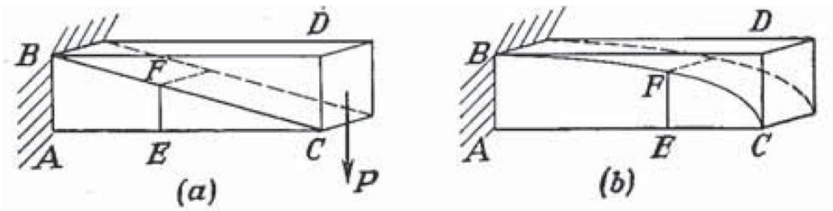


Galileo Galilei (1564-1642)

- End-supported beams are most sensitive to loads applied at the middle of the span; Bending moment is proportional to *ab*; The cross-sections near supports can be reasonably reduced.



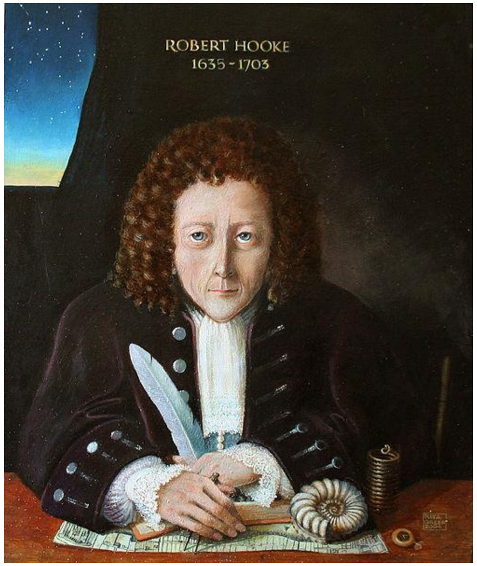
- Gave a complete derivation of a cantilever beam of equal strength: $EC:AC=(EF)^2:(AB)^2$



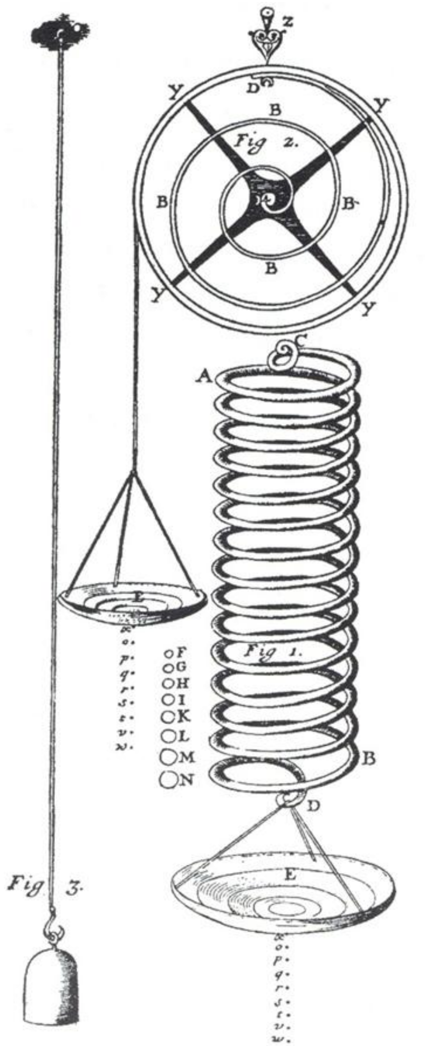
- Hollow beams are employed in art and still more often in nature (bones of birds, stems of straw…)

Robert Hooke (1635-1703)

- English mechanic and experimenter
- Thought that light was composed of waves.
- Contemporary and frequent adversary of Newton.
- First to study solids microscopically.《显微术》，1665.
- First to study the elastic properties of materials, foundation of further developments of mechanics of elastic bodies.
- Considered the deformation of longitudinal fibers of beams: the fibers on the convex (concave) side are extended (compressed) during bending.



Robert Hooke (1635-1703)

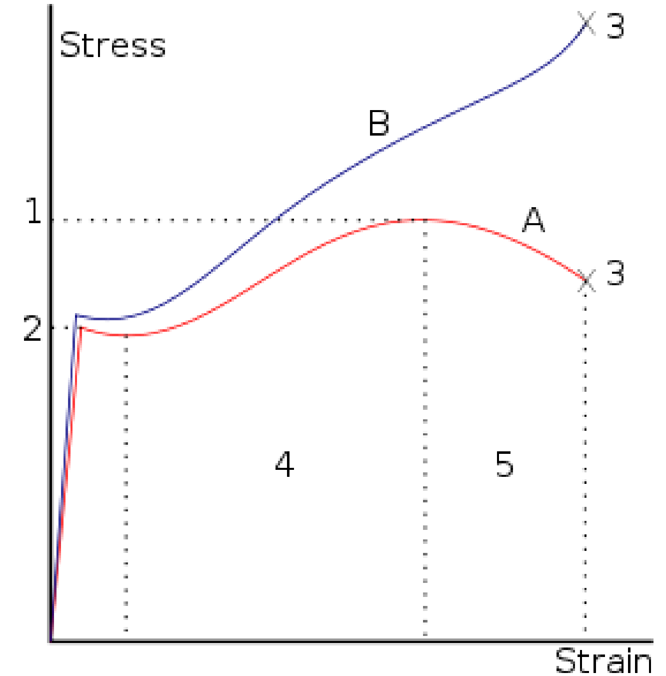


- Discovered Hooke's Law for a springing solid.
- Hooke's Law for a spring is similar to Hooke's Law for strength of materials:

$$F = -kx$$

$$\sigma = E\,\varepsilon$$

E = modulus of elasticty (property of the specific material)



Stress–strain curve for low-carbon steel. Hooke's law is only valid for the portion of the curve between the origin and the yield point(2).

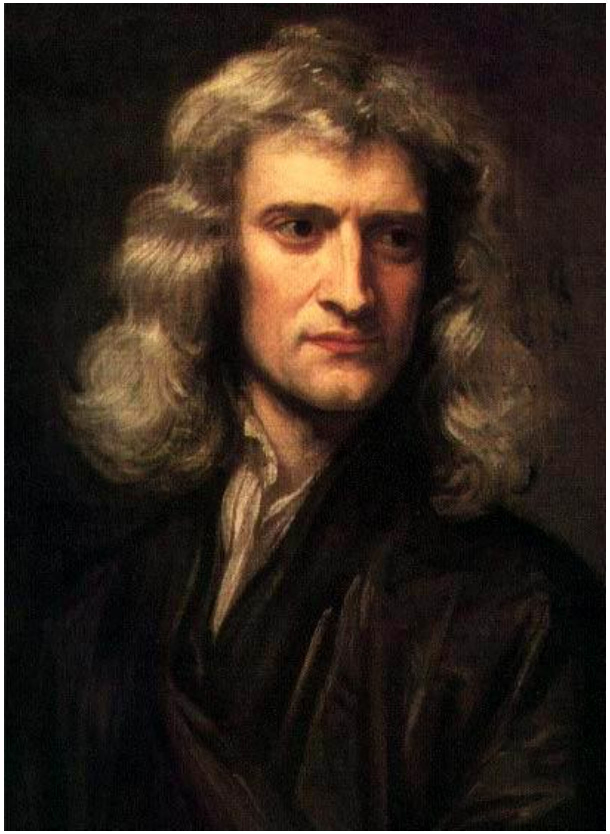
1. Ultimate strength
2. Yield strength-corresponds to yield point.
3. Rupture
4. Strain hardening region
5. Necking region. A: Apparent stress (F/A_0)
B: True stress (F/A)

郑玄 (公元127-200)

- 东汉经学大师、教育家。
- 郑玄的弓 vs. 胡克的弹簧
- Hooke's Law established the notion of linear elasticity, but not yet in a way that was expressible in terms of stress and strain.
- 每加物一石，则张一尺。
  Tension is proportional to the stretch.
- UT TENSIO, SIC UIS (as is the extension, so is the force).

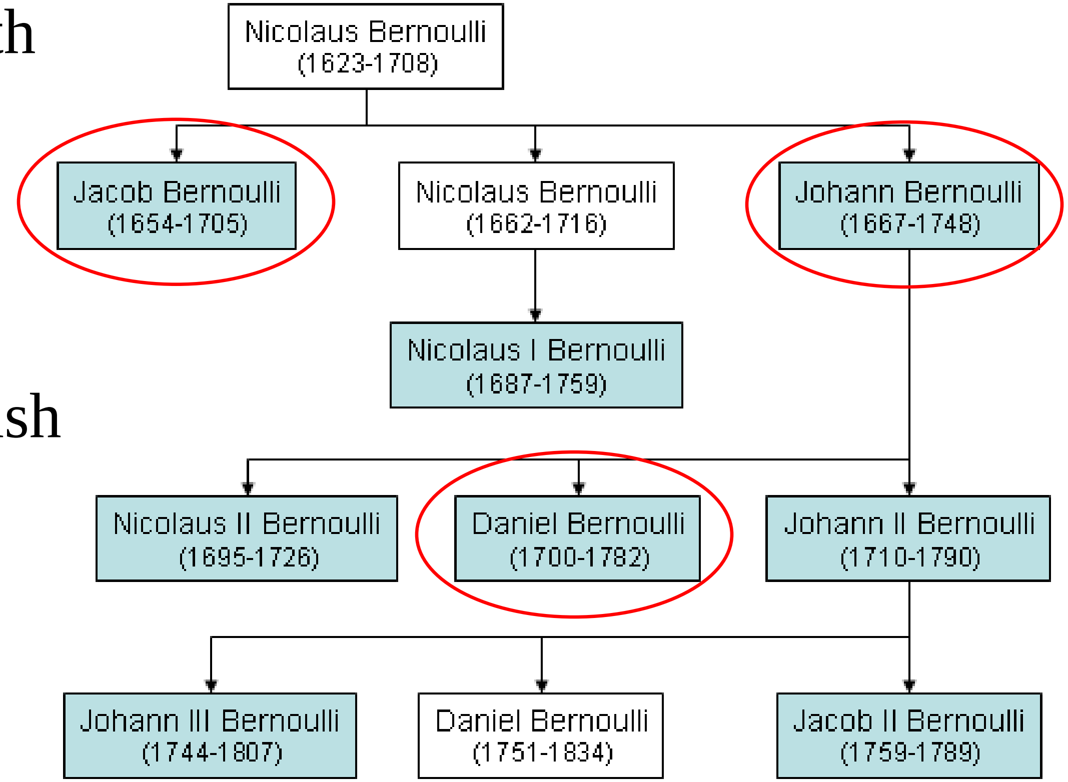
Sir Isaac Newton (1642-1727)

- 《自然哲学的数学原理》，1687
- Discovered laws of inertia, action and reaction of forces, and $F = m\,a$, known as Newton’s Laws, which are the foundations of rigid body (Newtonian) mechanics.
- Independently invented calculus with Leibniz and also worked in optics and thermodynamics.
- Contributed greatly to developing the scientific methods that are accepted today, *yet gave all credit to God for enabling him to make his great discoveries.*



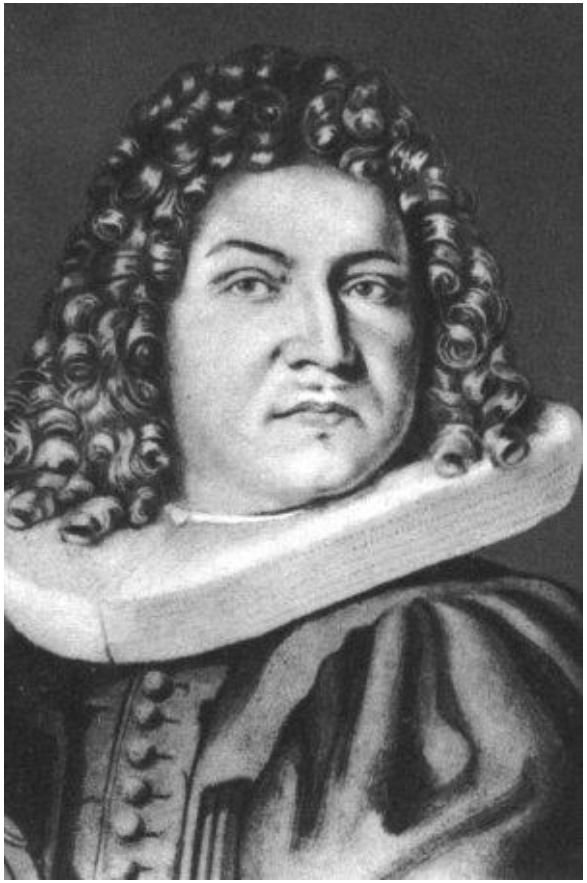
The Bernoulli Family

- The most predominant math family of Europe
- Originally from Holland with strong Calvinism religion and fled to Switzerland to avoid Spanish religious persecution.
- This family was not math oriented, they had a spice business in Bale.
- The uniqueness of this particular family is a stubborn streak which brought devastation to the family life



The Bernoulli Family

- **Jacob Bernoulli (1654-1705)**
- First analyzed beams from the point of view of deflection using calculus.
- First proposed the concept of force per unit area (stress) and related it to relative extension (strain).
- Cross-sections of beams remain planar under bending (Bernoulli Beams).
- Unfortunately treated the lower boundary of a beam as its neutral interface. This is finally fixed by Claude-Louis Navier in 1826.



Jacob Bernoulli (1654-1705)

- Bending of a cantilever beam (1694, 1705):

➢ Focused on beam deflection.

➢ Used calculus.

➢ Proposed plane hypothesis and normality between cross-sections and neutral interface.

➢ Lower boundary of the cantilever was taken as the neutral interface, resulting in tensile stresses for almost all longitudinal layers. (wrong!) Shear forces were neglected.

➢ Limited engineering usefulness since the D.E. was not simplified for small deflections. Curvature is hard to measure!

$$C \cdot \frac{1}{\rho} = Px \qquad C = \frac{Ebh^3}{3}$$

Leonard Euler (1707-1783)

- Swiss mathematician.
- Published over 800 papers, despite completely losing his eyesight in 1766!
- First to use the concept of strain energy.
- Developed the mathematical theory for columns and deformable bodies using differential equations long before being experimentally verified.
- Also developed the mathematics for rotating coordinate systems, which make stress transformation possible.

Leonard Euler (1707-1783)

- Bending of a cantilever beam (1744)
  - Cited the work by Bernoulli: $1/\rho \propto M$
  - From the mathematical point of interest.
  - Used variational approach: seeking the minimum of an integral, often relevant to energy
  - Made simplifications w.r.t. small deflections.
  - Wrong on the proportionality h^2. (The credit is still given to Jacob Bernoulli.)

$$C \cdot \frac{y''}{\left(1+y'^2\right)^{\frac{3}{2}}} = Px$$

$$C = \frac{Pl^2\left(2l-3f\right)}{6f} \approx \frac{Pl^3}{3f} \cancel{\propto bh^2}$$

- Stability of Columns
  - 1757，《关于柱的承载力》
  - Critical axial load of a beam:

$$Cy'' = -Py \Rightarrow P = \frac{C\pi^2}{4L^2}$$

  - This problem was revisited by Joseph Louis Lagrange（1736−1813）《柱的形状》

- Large deformations
- Beams of varying sections
- Curved beams

Charles-Augustin de Coulomb (1736-1806)

- Most known for Coulomb’s Law.
- Force of dry friction, 1781《简单机械》。
- Analyzed internal forces of beams using static equilibrium.
- Established correct concept on the transverse stress distribution on beam cross-sections.
- Invented torsional pendulum for measuring rigidity of circular wires in terms of the rotation of a connected circular cylinder. A very precise method!

Joseph Louis Lagrange (1736-1813)

- French mathematician and scientist.
- Established the calculus of variations and the theory of differential equations.
- Lagrangian mechanics used calculus to deduce the whole of mechanics from fundamental principles.
- "I have always observed that the pretensions of all people are in exact inverse ratio to their merits; this is one of the axioms of morals."

Claude-Louis Navier (1785-1836)

- Bending of beams (1826):

- First correctly identified the position of neutral interfaces for bending beams.（**中性层通过截面的形心，1826年**《力学在机械与结构方面的应用》，巴黎综合工科学校授课讲义，系统讲述材料力学第一本书。）

$$EI \cdot \frac{1}{\rho} = M$$

- 俄罗斯铁路工程师茹拉夫斯基（1821-1891）于1855年得到横力弯曲时的切应力公式。

- 1855年，俄罗斯工程师别斯帕罗夫的《用初等方法求解关于材料力学与结构稳定性的问题》在彼得堡出版，介绍弯矩图，并开始使用弯矩图求解梁问题。至此，梁的线性理论问题得到完全解决。

Strength of Materials is done in the mid-19th century.

Content Description

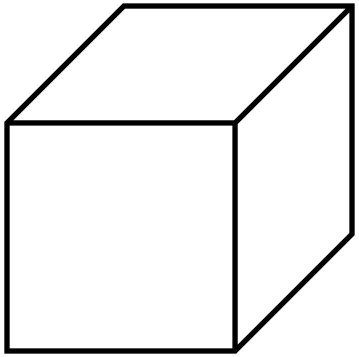
Analyze the response of rest bodies and structures to forces.

- Statics: The Analysis of Bodies at Rest.
- *Dynamics: The Analysis of Bodies in Motion.*
- Strength of Materials: The Analysis of Deformable Bodies, either statically determinate or not.
- *Structural Mechanics: The Analysis of Deformable Structures, which are often statically indeterminate.*

Content Description

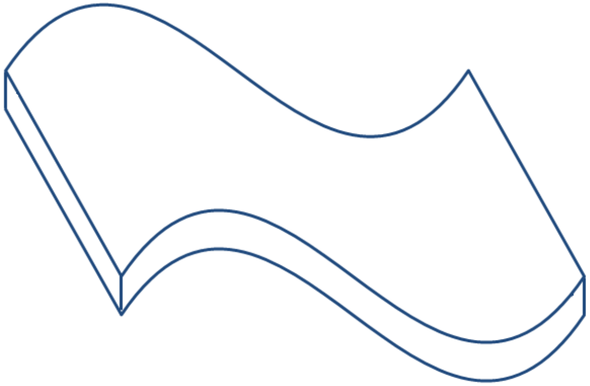
- *Introduction to Statics*
- Introduction to the concepts of stress, deformation, and strain in solid materials.
- Development of basic relationships between loads, stresses, and deformation and deflections of structural and machine elements such as rods, pressure vessels, and beams.
- Load-carrying capacity of these elements under tension, compression, shearing, torsion, bending, and their possible combinations.

Classification of Bodies by Dimension

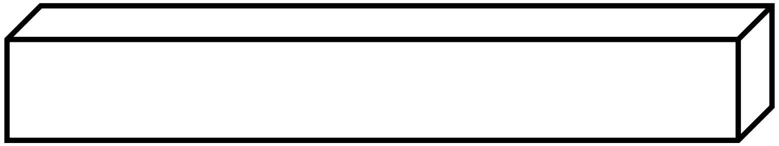


Bodies: three dimensions are equivalent.

Plates: two dimensions are far greater than the third. No curvature exists in the median plane of the plate.



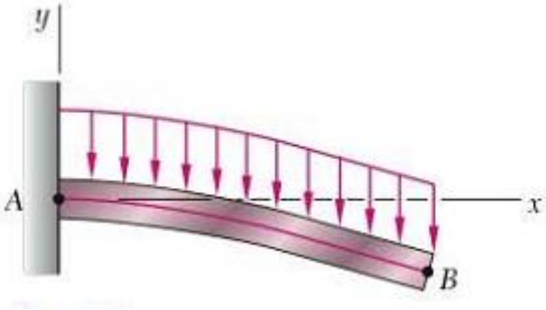
Shells: two dimensions are far greater than the third. At least one principal curvature is nonzero in the median plane of the plate.

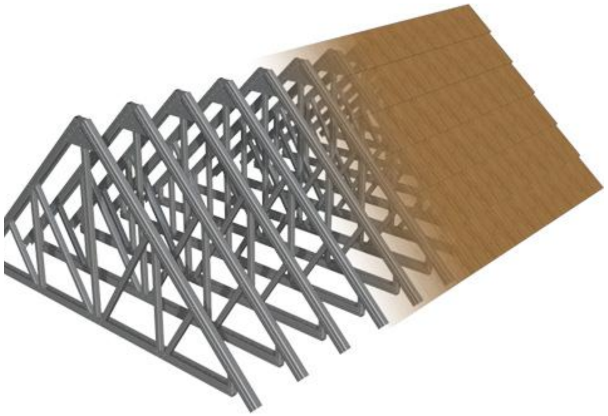


Bars: one dimension is far greater than the others. The bar axis may be either straight or curved.

Classification of Bodies by Dimension+Deformability

- Particle: Point Mass
- Rigid Body: Mass + Volume, but No Deformation
- Deformable Body: Mass + Volume + Deformation
- Deformable Structure: Structure + Deformation
- *Note that this course does not deal with bodies/structures in motion.*

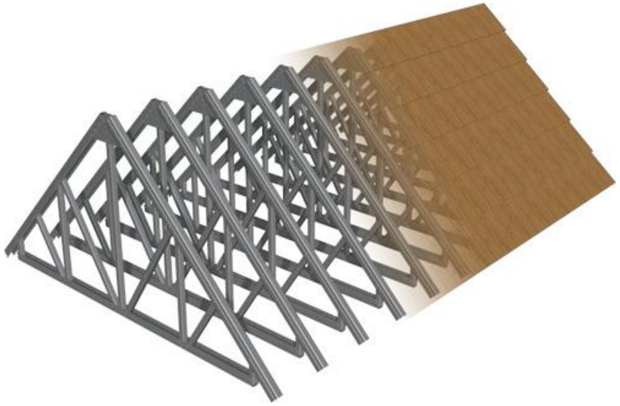




Mass Points Rigid Bodies Deformable Solids Deformable Roof Trusses

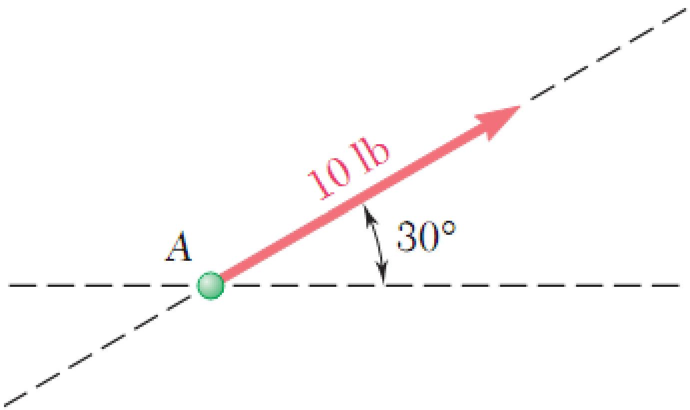
Structures Composed of Prismatic Bars

- Space (3-D) Structures: Bars/loads don't belong to the same plane
- Plane (2-D) structures: Bars/loads belong to the same plane
- Bar, Column, Beam
- Curved Beam (arch)
- Plane Frame
- Truss
- Suspended-cable structure



Loads: Force and Moment

- **Force:** action of one body on another; characterized by its point of application, magnitude, line of action, and sense.

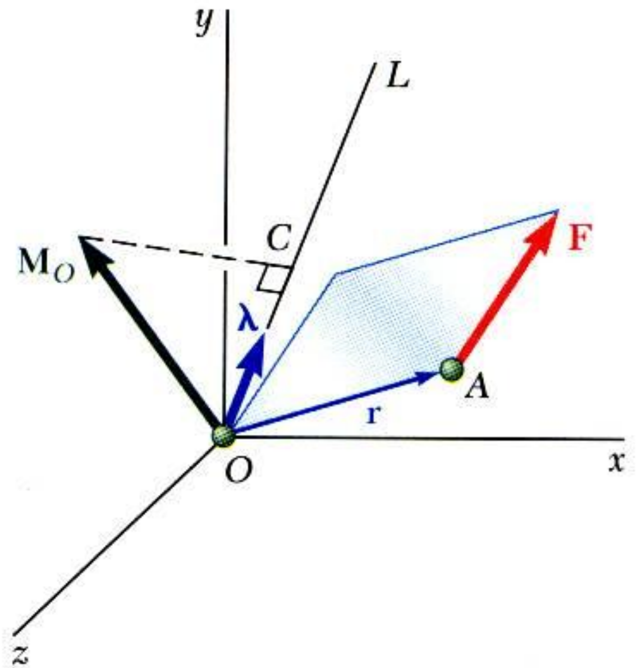


- **Moment:** $\boldsymbol{M_O}$ of a force $\boldsymbol{F}$ applied at the point A about a point O,

$$\vec{M}_O = \vec{r} \times \vec{F}$$

- **Scalar moment:** M_{OL} about an axis $\boldsymbol{OL}$ is the projection of the moment vector $\boldsymbol{M_O}$ onto the axis,

$$M_{OL} = \vec{\lambda} \cdot \vec{M}_O = \vec{\lambda} \cdot \left(\vec{r} \times \vec{F} \right)$$



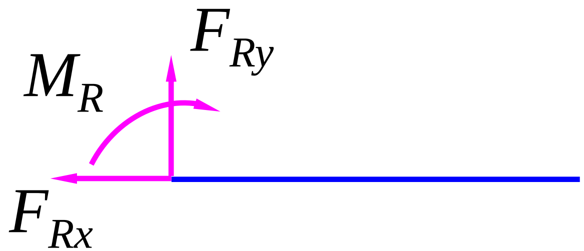
- **Thermo-loads (Temperature)**

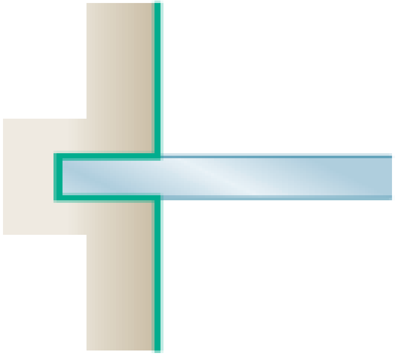
Classification of Loads

Criteria	**Classification**
Exerting Objects	Active/restraint Loads
Size of Acting Scope	Concentrated/Distributed Loads
Dimensions of Distributed Loads	Volume/Surface/Line Loads
Acting Period	Instantaneous(Live)/Constant Loads
Constancy w.r.t. Time	Static/Dynamic Loads
Characteristics	External Loads → Internal Forces

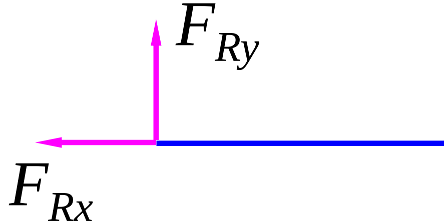
Restraints and Reactions

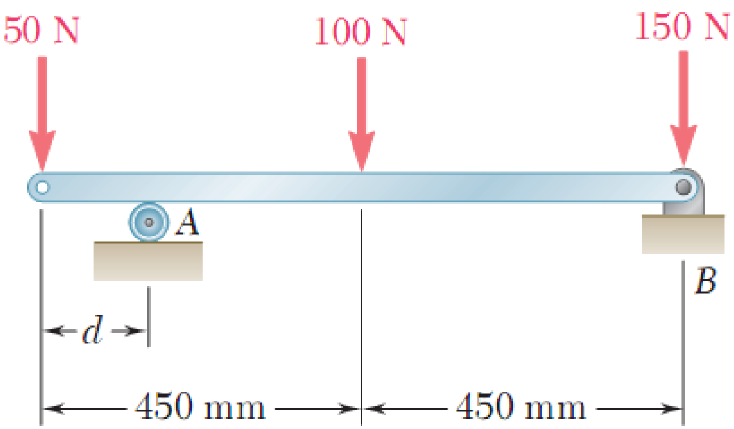
① Fixed support





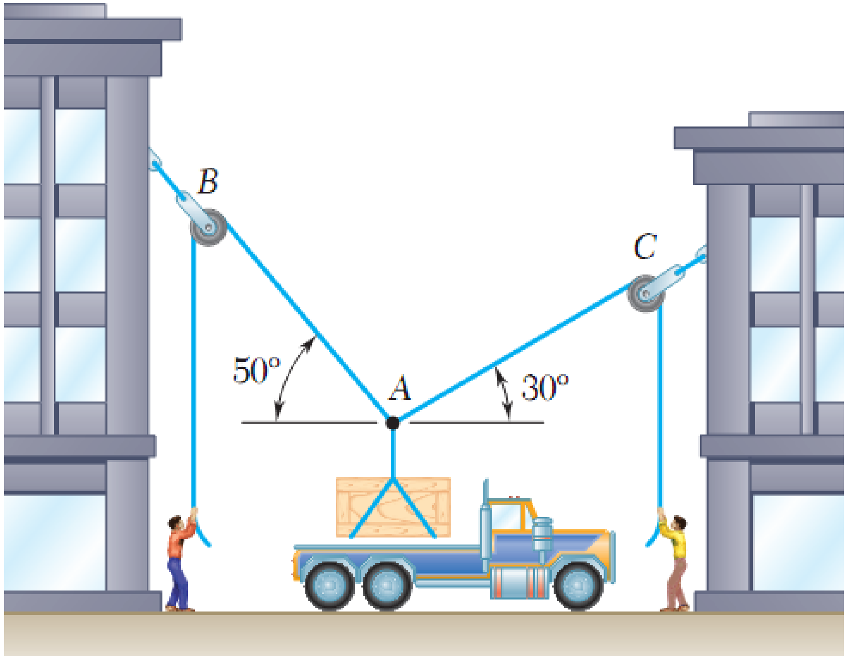
② Hinged or pinned support

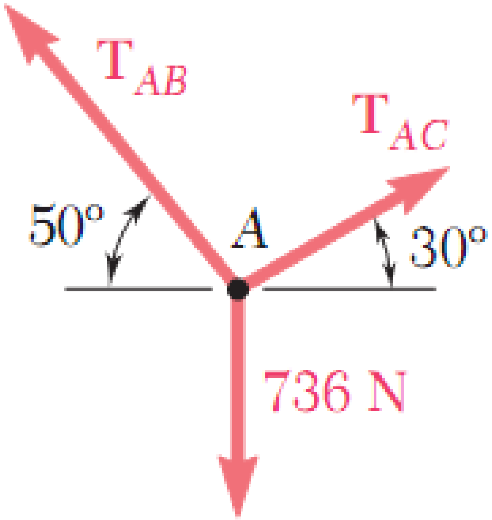




③ Roller support

Space Diagram vs. Free-Body Diagram





Space Diagram:

A sketch showing the physical conditions of the problem.

Free-Body Diagram:

A sketch showing only the forces on the simplified body/structure.

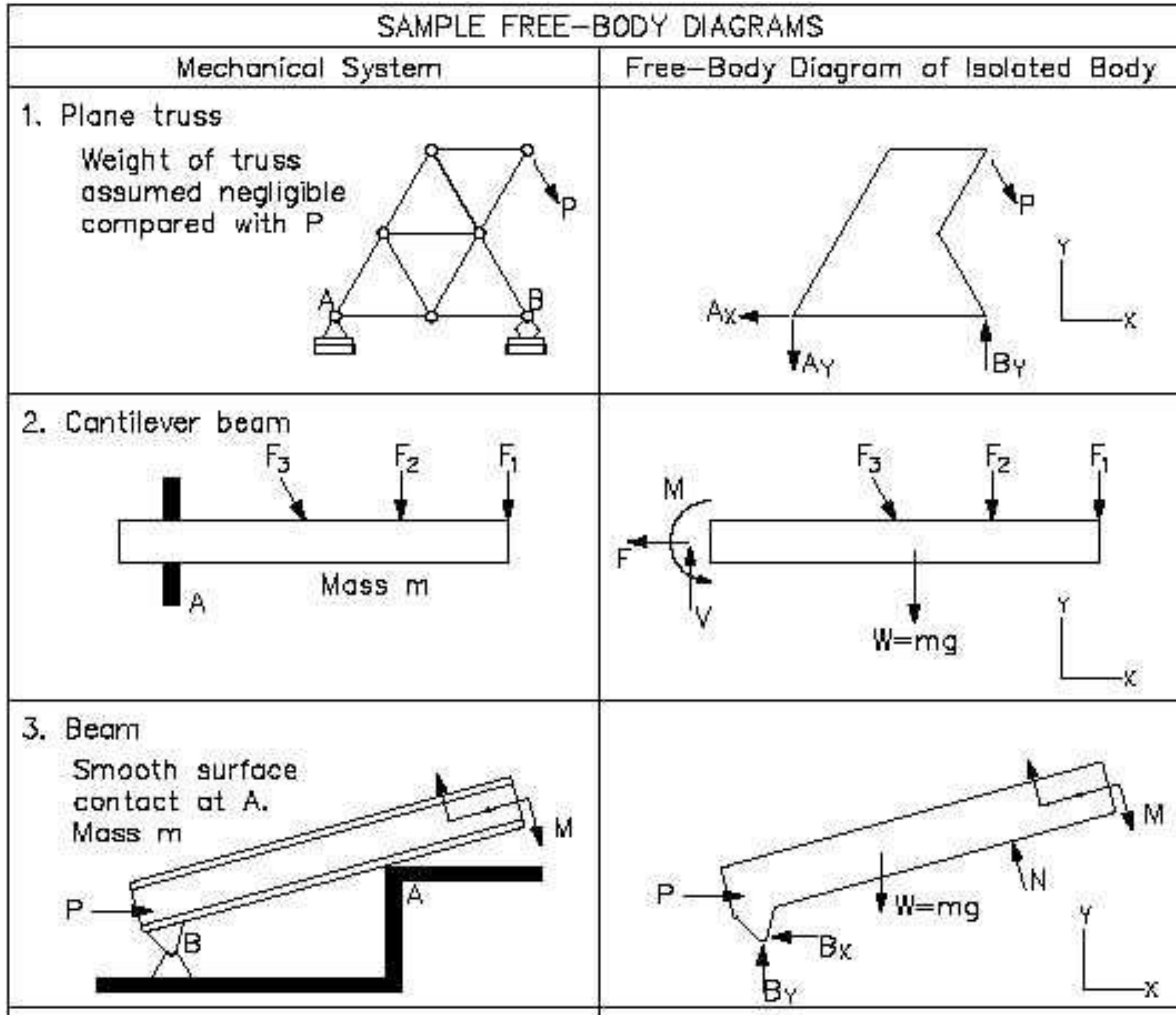
Space Diagram vs. Free-Body Diagram

Simplification Convention:

1. Simplification of body/structure (plane, line & joints)
2. Simplification of supports (clamped/pinned/roller)
3. Simplification of loads (concentrated/distributed, force/moment, surface/line)

Simplification Procedure:

1. Separation of body/structure
2. Determining external loads
3. Determining reaction forces

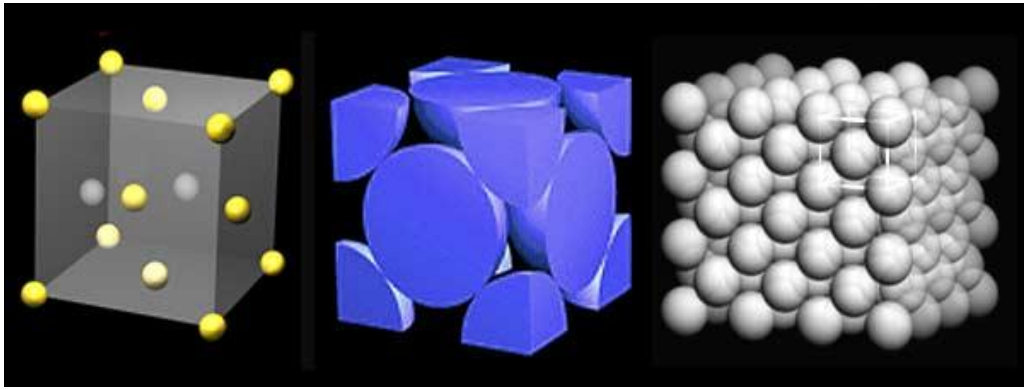


Assumptions of Strength of Materials

- Continuity
- Homogeneity
- Isotropy
- Small Deformation Linear Elasticity

Continuity

- Matter fills up the whole space of a solid defined by its volume.
- Neither vacancies can be produced nor more materials can be added under normal working conditions.
- Arbitrary section or volume element can be extracted for force or deformation analysis.



Face-centered cubic lattice

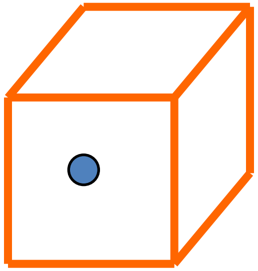
$$4r = a\sqrt{2} \Rightarrow r = \frac{\sqrt{2}}{4}a$$

$$V_{matter} = \frac{16}{3}\pi r^3 = \frac{16}{3}\pi \frac{2\sqrt{2}}{64}a^3 = \frac{\pi}{3\sqrt{2}}a^3$$

$$\Rightarrow \frac{V_{matter}}{V} = \frac{\pi}{3\sqrt{2}} \approx 74\%$$

Homogeneity

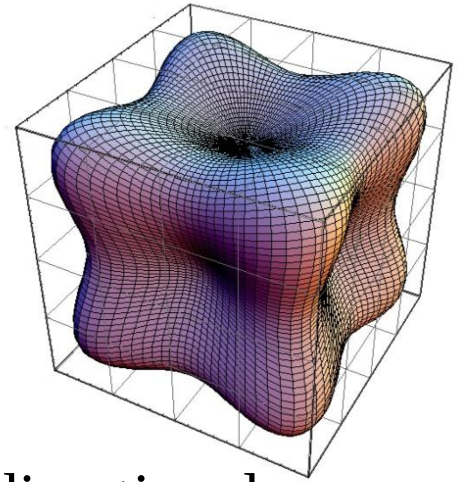
- Macroscopic material properties can be represented by those of any arbitrary representative volume element (RVE).
- The minimum size of a RVE depends on material types, i.e. $0.1 \times 0.1 \times 0.1$ mm for metals; $10 \times 10 \times 10$ mm for concrete
- A RVE should be composed of at least the number of "elementary entities" (usually atoms or molecules) in one mole, i.e. 6.022×10^{23}



A representative volume element in a solid.

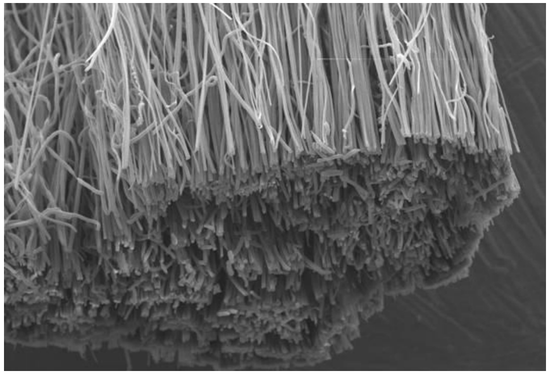
Isotropy

- Mechanical properties of materials are independent of directions.
- Mechanical properties of materials are taken as the statistical average of those along every direction.
- Strong transversely isotropic materials such as wood and fiber reinforced composites are still viewed as non-isotropic materials.



The directional dependence of Young's modulus

Cross section of woods.

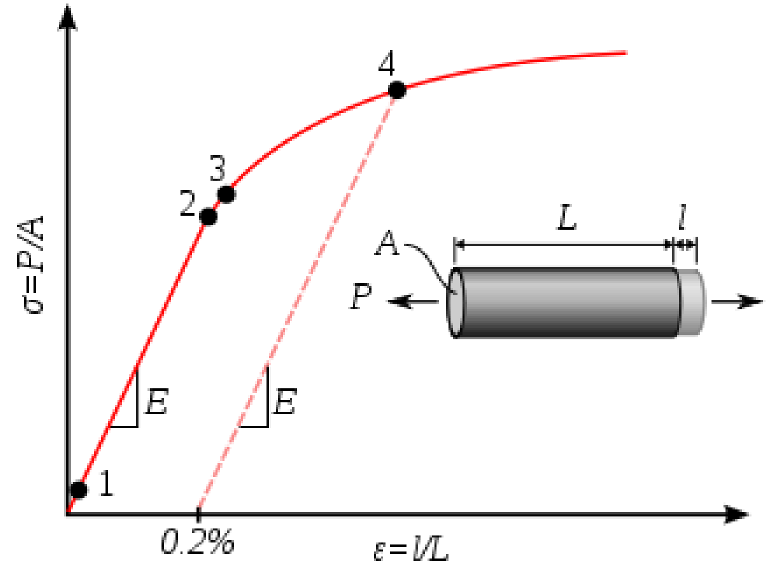


Carbon fiber reinforced composites

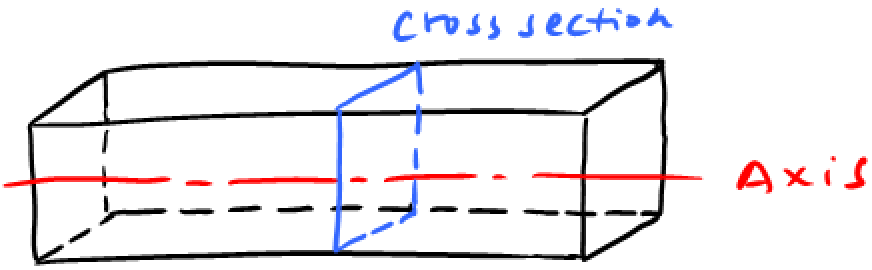
Small Deformation Linear Elasticity

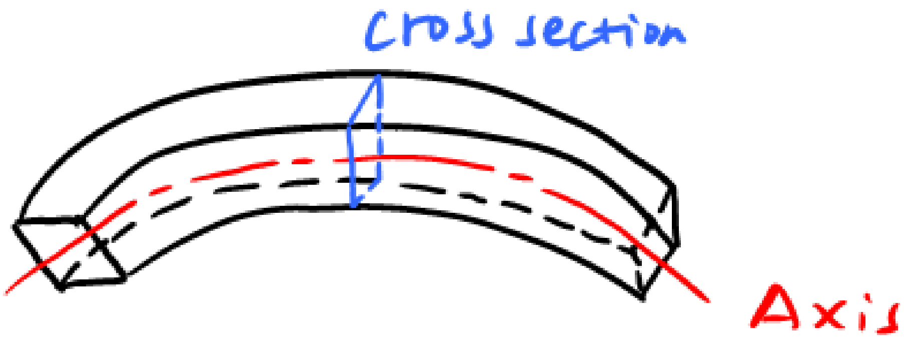
- **Small Deformation:** deformation of structural elements under mechanical loads are negligible compared to their original size.
- Under small deformation, analysis of force and deformation can be based on a structure's size and shape prior to deformation.
- **Elasticity:** a structural element can restore to its original size and shape upon the removing of its external loading.
- **Linear Elasticity:** deformation is linearly proportional to load.

Stress-strain curve showing yield behavior. 1. A point within proportionality. 2. Proportionality limit. 3. Elastic limit (initial yield strength). 4. Subsequent yield strength.



Geometric Characteristics of Prismatic Bars

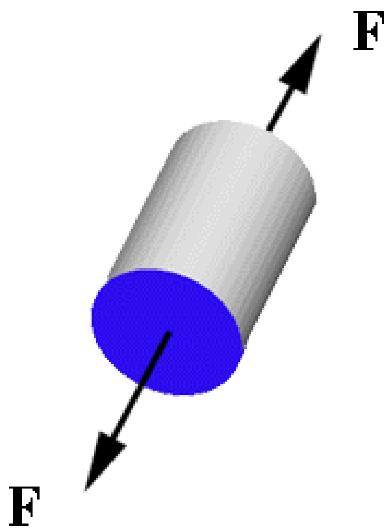


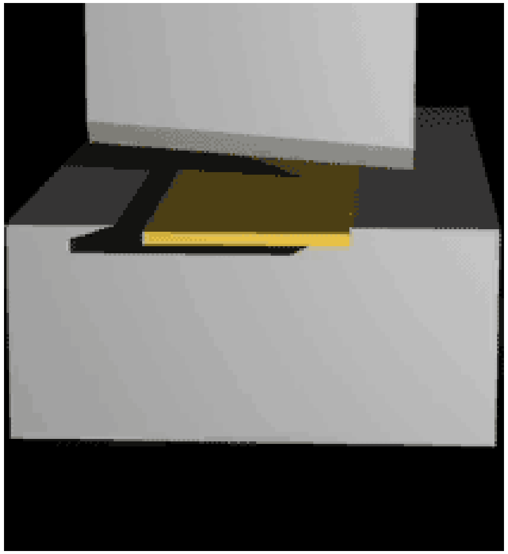


- **Geometric Factors:**
  - Cross section (may vary)
  - Axis (may be curved)
  - Axis is perpendicular to cross sections

- We are mainly concerned with straight and equal cross-sectional prismatic bars in this course. The theory, however, can also be applied to:
  - Curved bars with small curvature
  - Slightly varying cross-sectional bars

Forms of Deformation of Prismatic Bars





Tension

- Load: equal and opposite force along bar axis
- Deformation: elongation along axis

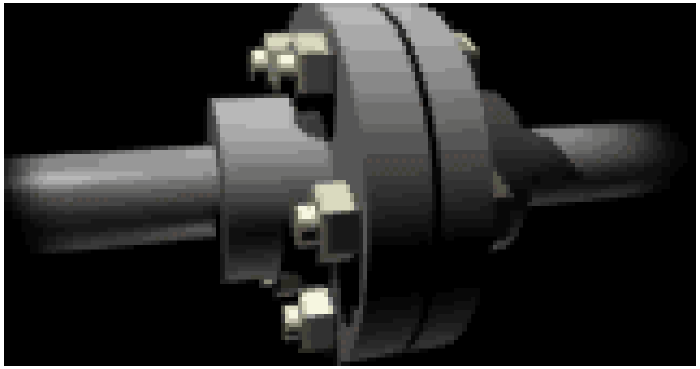
Compression

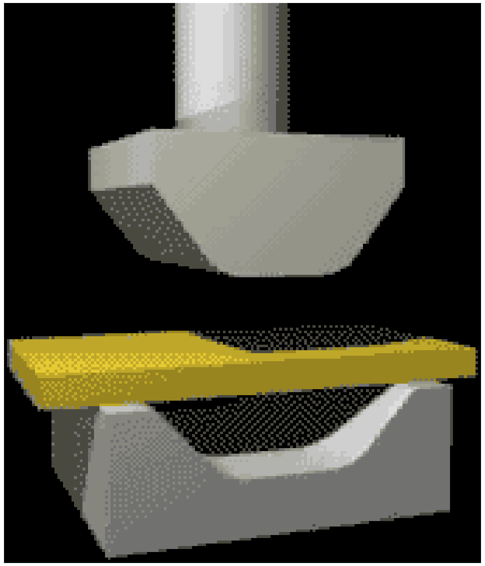
- Load: equal and opposite force along bar axis
- Deformation: contraction along axis

Shearing

- Load: equal and opposite force perpendicular to axis
- Deformation: relative shear of adjacent cross sections

Forms of Deformation of Prismatic Bars





Torsion

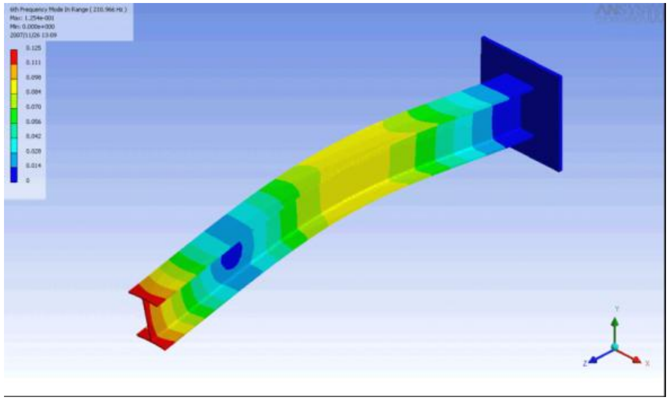
- Load: equal and opposite moment applied in planes perpendicular to bar axis
- Deformation: relative rotation of adjacent cross sections around axis

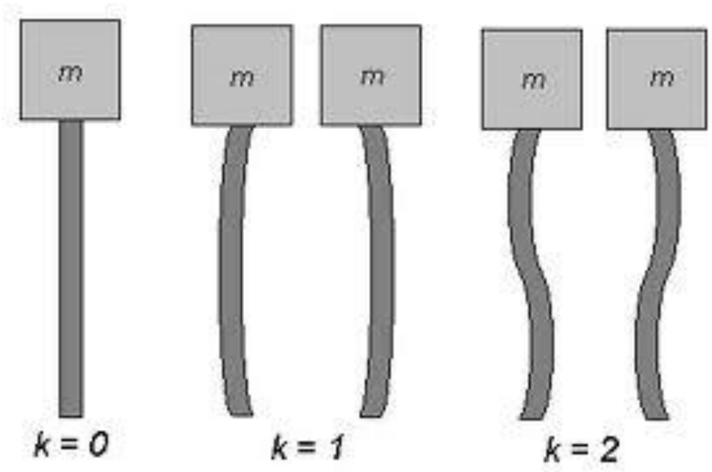
Bending

- Load: equal and opposite moment applied in planes containing the bar axis
- Deformation: relative rotation of adjacent cross sections around the axis perpendicular to bar axis

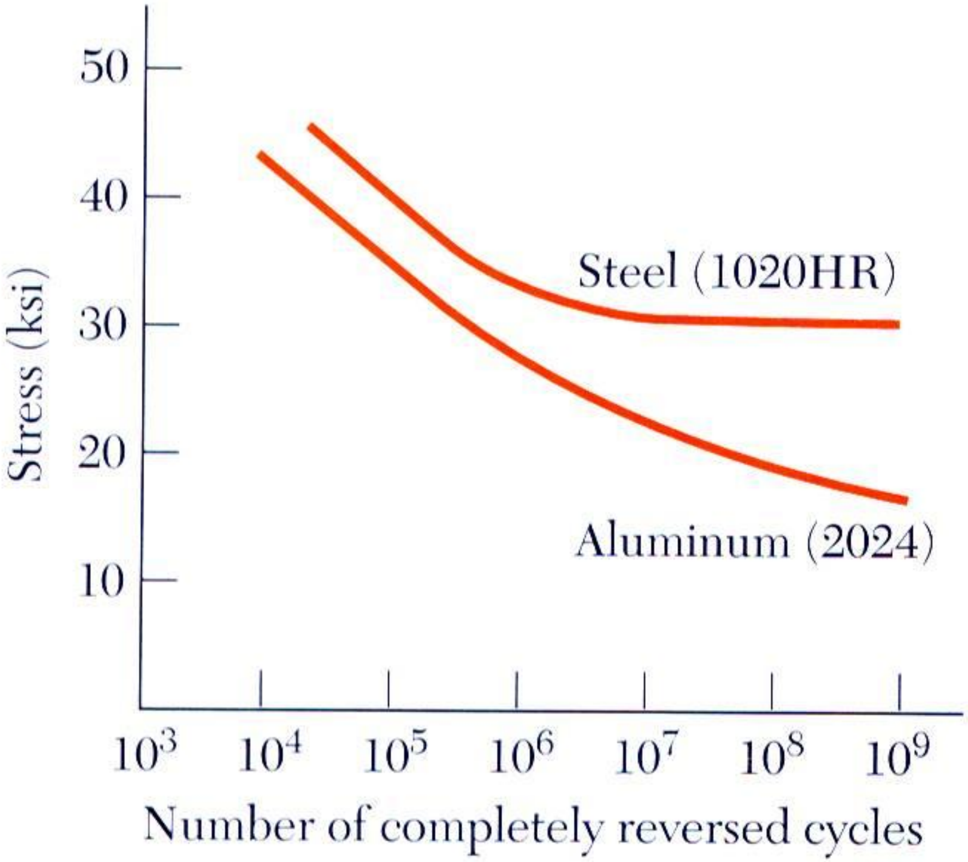
Requirements of A Structural Element

- **Strength Condition:** structural components should not fracture or yield under external loads
- **Stiffness Condition:** maximum deformation should be under the allowable extent
- **Stability Condition:** the equilibrium of a structural element should be a stable one





Fatigue



- Fatigue properties are shown on σ-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.

Course Objectives

- The main objective of the study of strength of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.
- Both the analysis and design of a given structure involve the determination of stresses and deformations.

Topics Covered (Statics)

- *Statics of Particles （质点静力学）*
- *Equivalent Systems of Forces （力系的等效）*
- *Equilibrium of Rigid Bodies （刚体的平衡）*
- *Analysis of Determinate Structures （静定结构的分析）*
- Centroids and Centers of Gravity （形心与重心）
- Moments and Product of Inertia （惯性矩与惯性积）

Topics Covered (Strength of Materials)

- Stress, Strain, Basic Elasticity, and Axial Loading（应力、应变、基本弹性理论、拉伸与压缩）
- Shearing & Bearing Stresses（剪切与挤压应力）
- Torsion（扭转）
- Bending Internal Forces（弯曲内力）
- Bending Stresses（弯曲应力）
- Bending Deflections（弯曲变形）
- Statically Indeterminate Structures（超静定杆件与结构）
- Stress States（应力状态）
- Strength Theory（强度理论）
- Combined Loadings（组合荷载）
- Stability of Columns（压杆失稳理论）
- Energy Methods（能量法）
- Dynamic Loading（动荷载）
- Cyclic Loading and Fatigue（循环荷载与疲劳）

Contents

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- Content Description（课程描述）
- Classification of Solid Bodies（固体的分类）
- Structures Composed of Prismatic Bars（杆系结构）
- Loads（荷载）
- Restraints and Reactions（约束与支反力）
- Space vs. Free-body Diagram（真实系统与受力简图）
- Assumptions of Strength of Materials（材料力学基本假设）
- Geometric Characteristics of Prismatic Bars（杆件的几何特征）
- Forms of Deformation of Prismatic Bars（杆件基本变形形式）
- Requirements of a Structural Element（构件基本要求）
- Fatigue（疲劳）
- Course Objectives（课程目标）
- Topics Covered（主要议题）