Statics of Particles

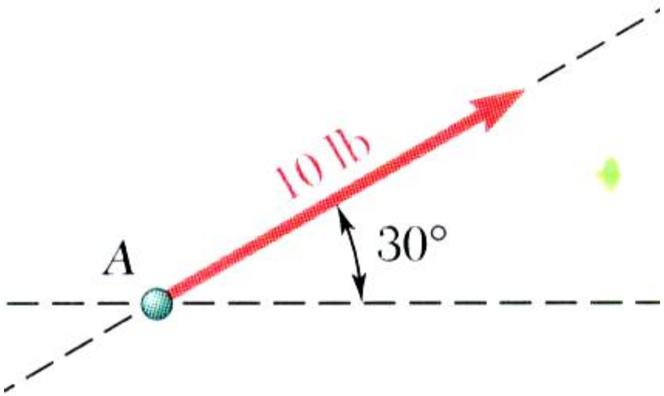
Contents

Introduction

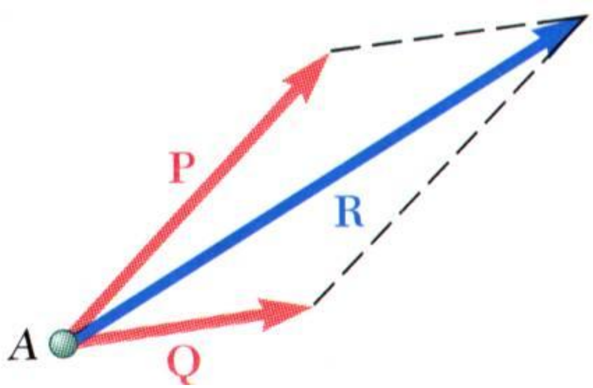
- The objective for the current chapter is to investigate the effects of forces on particles:
  - replacing multiple forces acting on a particle with a single equivalent or *resultant* force,
  - relations between forces acting on a particle that is in a state of *equilibrium*.

- The focus on *particles* does not imply a restriction to miniscule bodies. Rather, the study is restricted to analyses in which the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.

Resultant of Two Forces

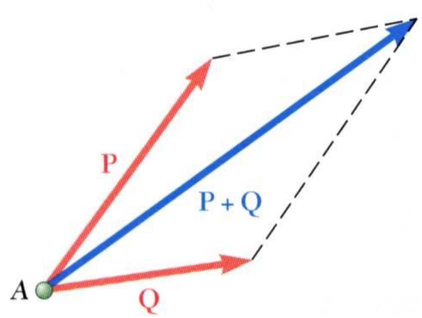


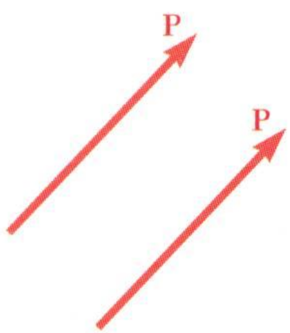
- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.

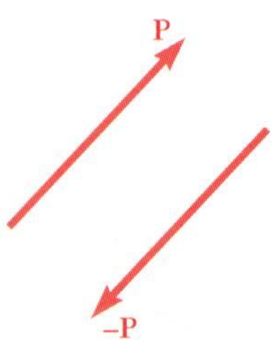


- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

Vectors

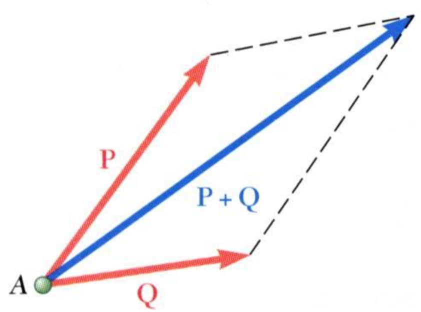


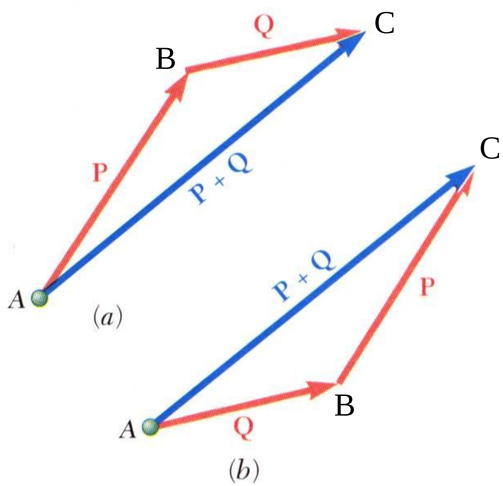


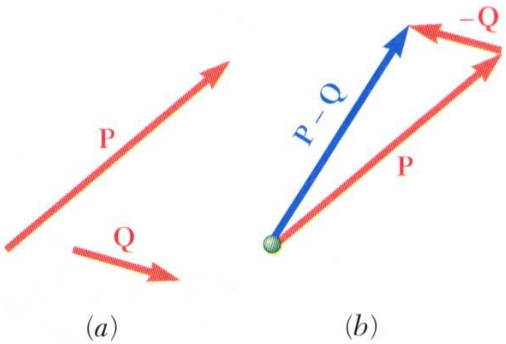


- *Vector*: parameter possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.
- *Scalar*: parameter possessing magnitude but not direction. Examples: mass, volume, temperature
- Vector classifications:
  - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
  - *Free* vectors may be freely moved in space without changing their effect on an analysis.
  - *Sliding* vectors may be applied anywhere along their line of action without affecting an analysis.
- *Equal* vectors have the same magnitude and direction.
- *Negative* vector of a given vector has the same magnitude and the opposite direction.

Addition of Vectors







- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

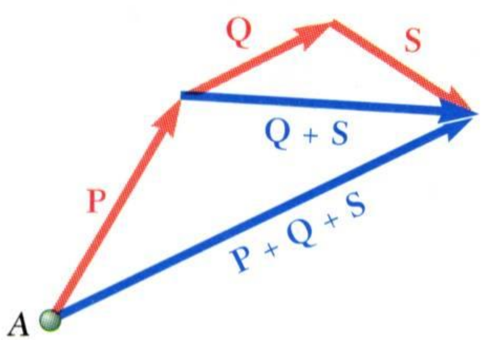
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{P}$$

- Vector addition is commutative,

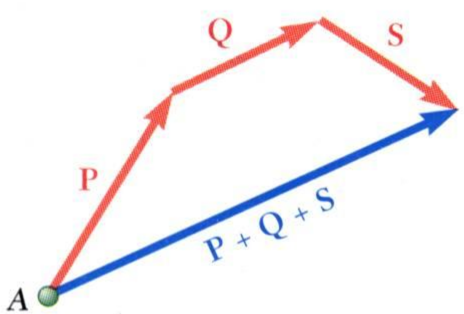
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Addition of Vectors

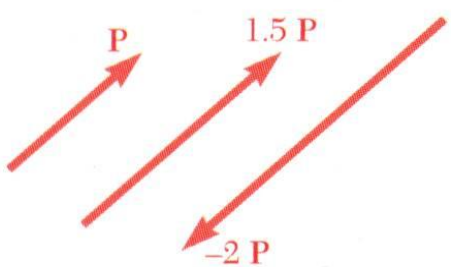


- Addition of three or more vectors through repeated application of the triangle rule



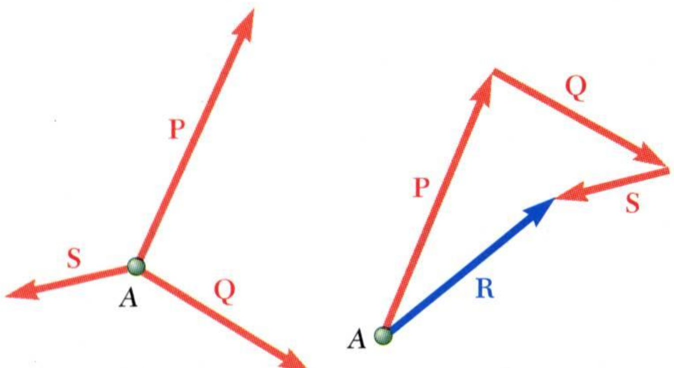
- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$

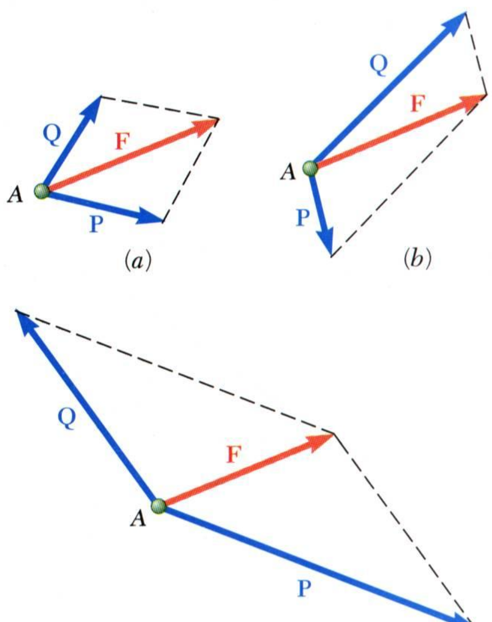


- Multiplication of a vector by a scalar

Resultant of Several Concurrent Forces



(a) (b)



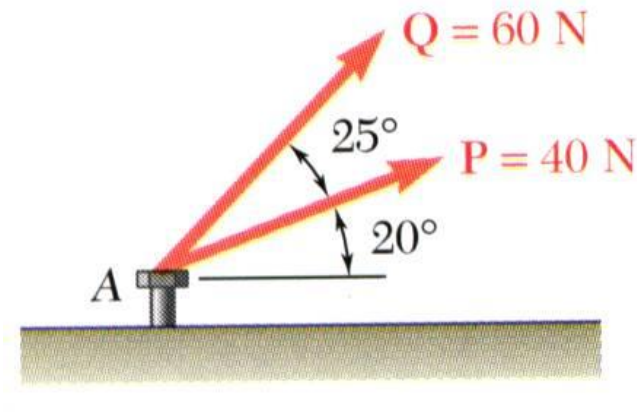
(c)

- *Concurrent forces*: set of forces which all pass through the same point.

  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.

- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Sample Problem

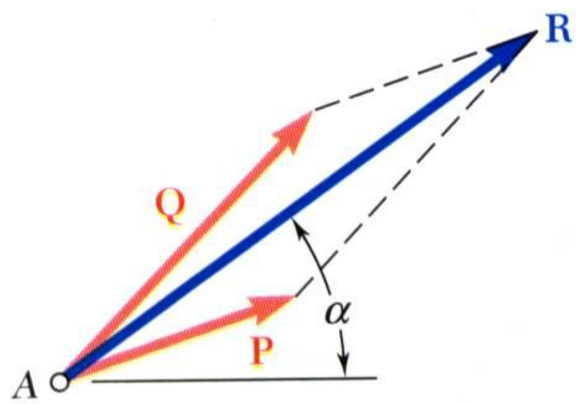


The two forces act on a bolt at *A*. Determine their resultant.

SOLUTION:

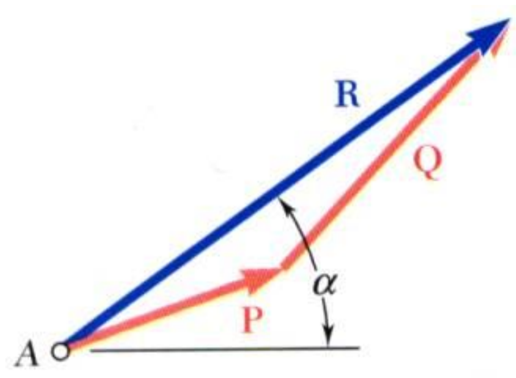
- Graphical solution - construct a parallelogram with sides in the same direction as **P** and **Q** and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the the diagonal.

- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.



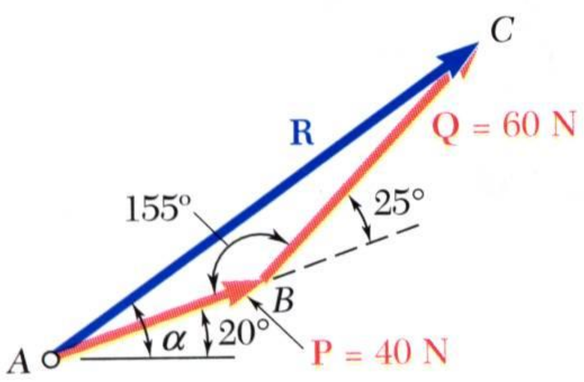
- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35^\circ}$$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35^\circ}$$



- Trigonometric solution - Apply the triangle rule.

From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\mathrm{N})^2 + (60\mathrm{N})^2 - 2(40\mathrm{N})(60\mathrm{N})\cos 155°$$

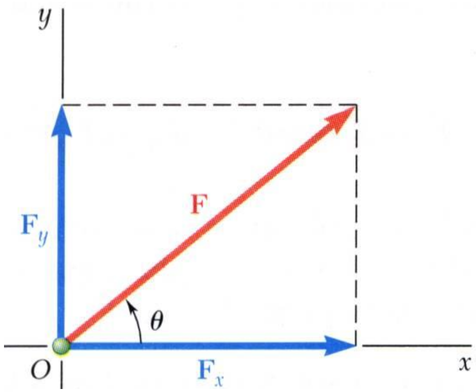
$$\boxed{R = 97.73\mathrm{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

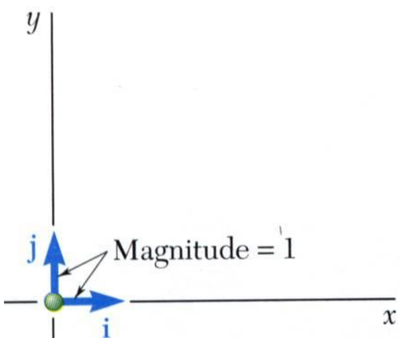
$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\mathrm{N}}{97.73\mathrm{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Rectangular Components of a Force: Unit Vectors

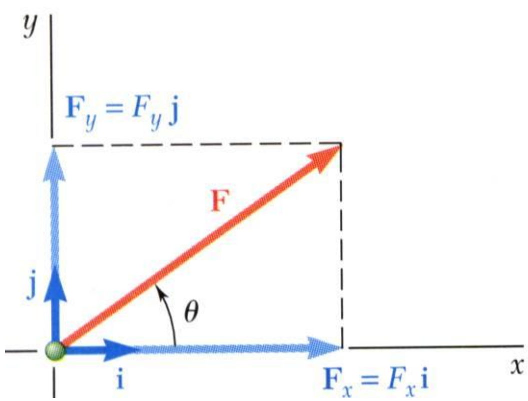


- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$



- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

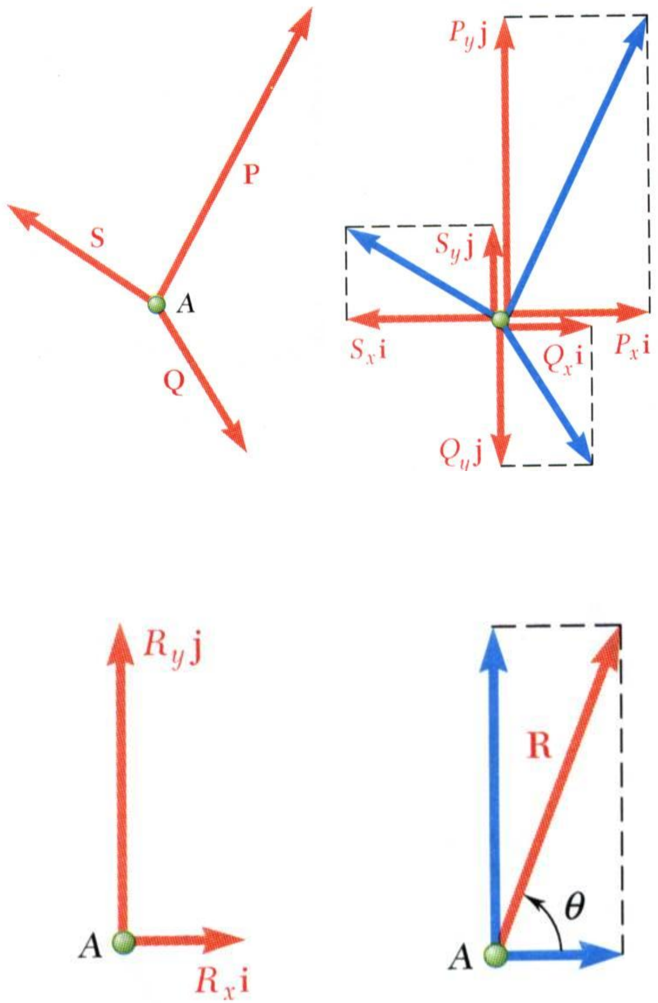


- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

Addition of Forces by Summing Components



- Wish to find the resultant of 3 or more concurrent forces,

  $$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

  $$\begin{aligned} R_x\vec{i} + R_y\vec{j} &= P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j} \\ &= \left(P_x + Q_x + S_x\right)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j} \end{aligned}$$

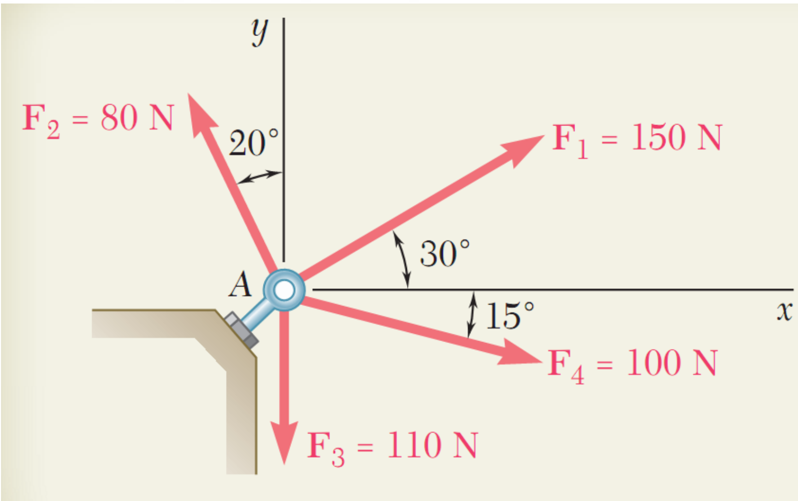
- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

  $$\begin{aligned} R_x &= P_x + Q_x + S_x \\ &= \sum F_x \end{aligned} \qquad \begin{aligned} R_y &= P_y + Q_y + S_y \\ &= \sum F_y \end{aligned}$$

- To find the resultant magnitude and direction,

  $$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

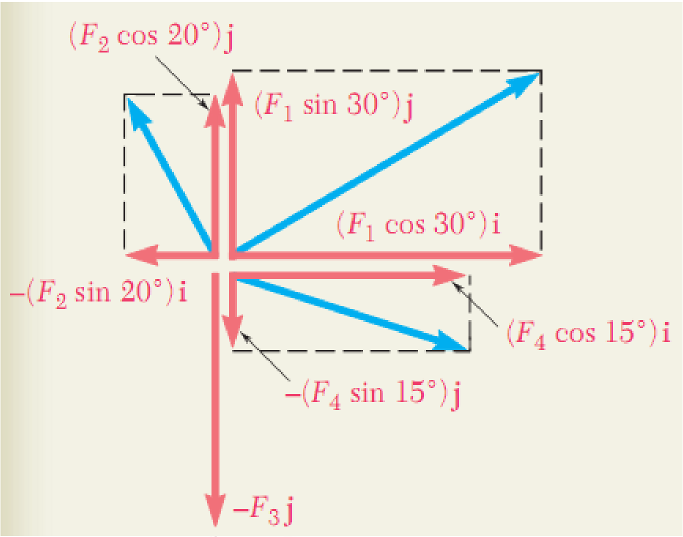
Sample Problem

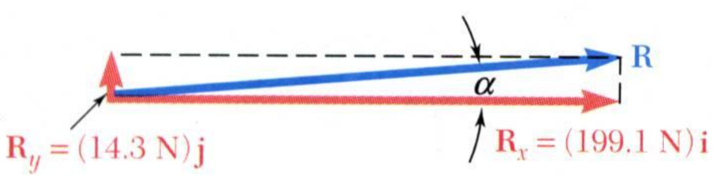


Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.





SOLUTION:

- Resolve each force into rectangular components.

$force$	mag	$x-comp$	$y-comp$
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

- Determine the components of the resultant by adding the corresponding force components.

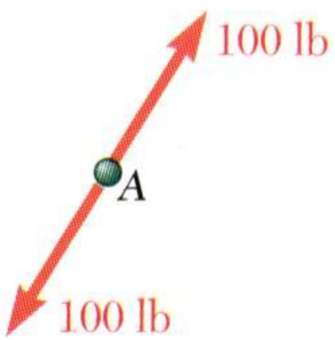
- Calculate the magnitude and direction.

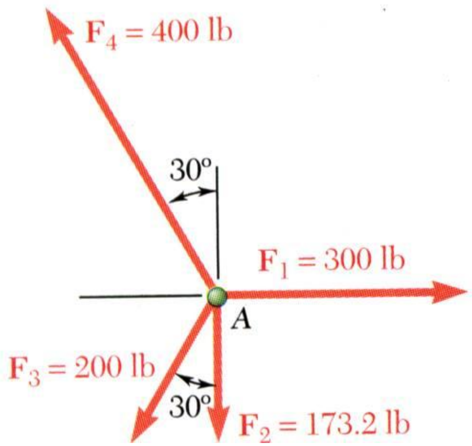
$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\,\mathrm{N}}$$

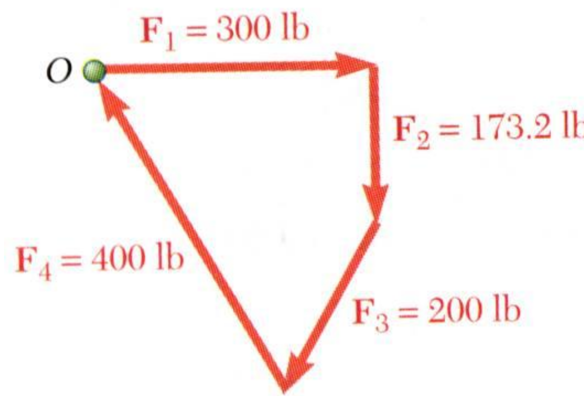
$$\tan\alpha = \frac{14.3\,\mathrm{N}}{199.1\,\mathrm{N}} \qquad \boxed{\alpha = 4.1^\circ}$$

Equilibrium of a Particle

- When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.
- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.







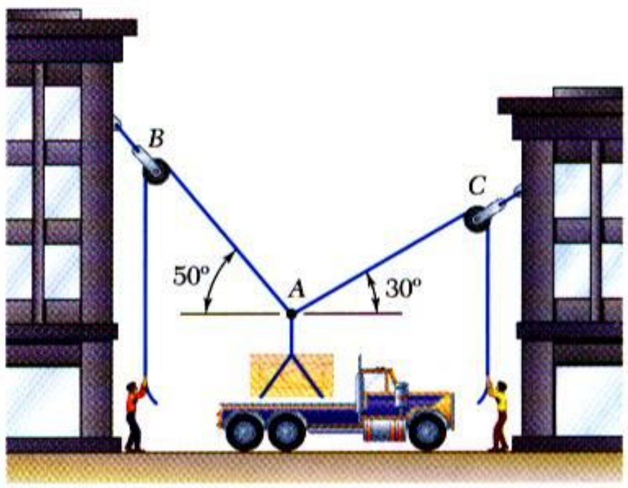
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense

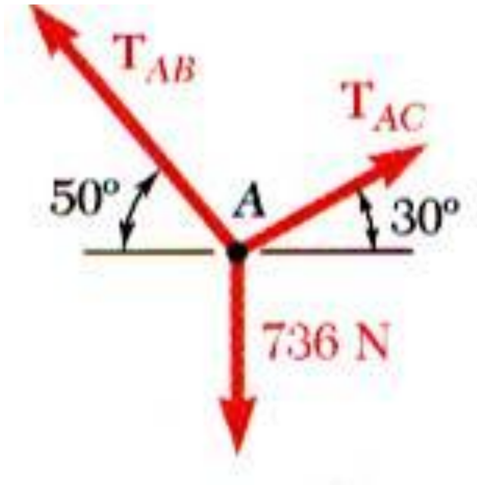
- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Free-Body Diagrams

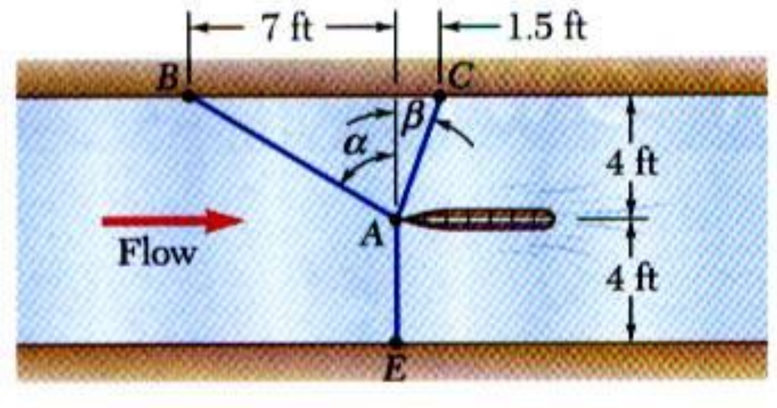




Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle.

Sample Problem

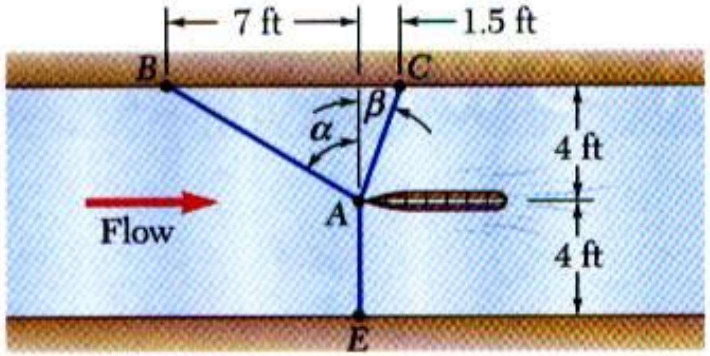


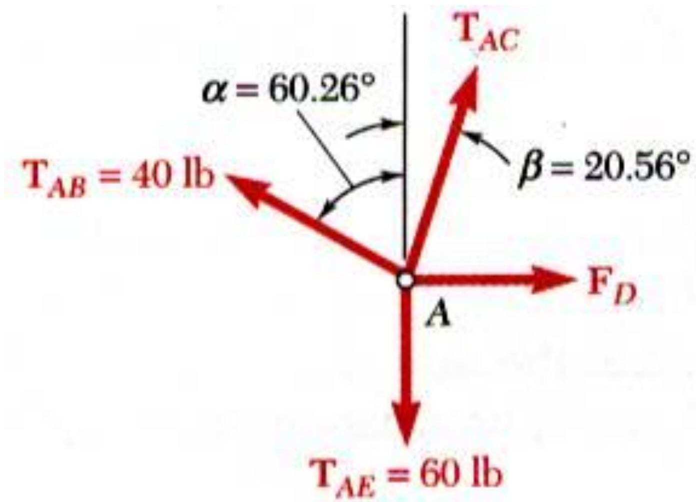
It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable *AB* and 60 lb in cable *AE*.

Determine the drag force exerted on the hull and the tension in cable *AC*.

SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.
- Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.
- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.





SOLUTION:

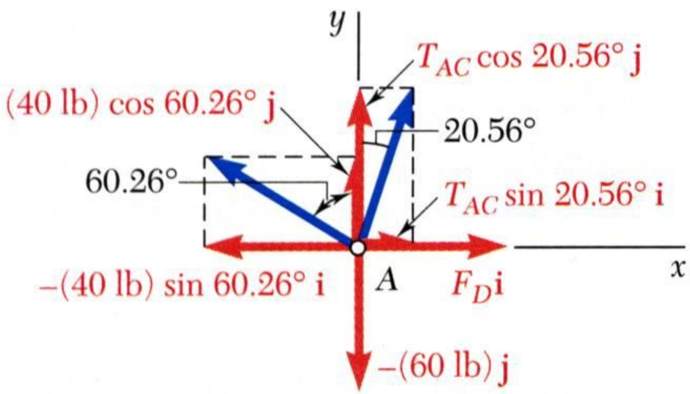
- Choosing the hull as the free body, draw a free-body diagram.

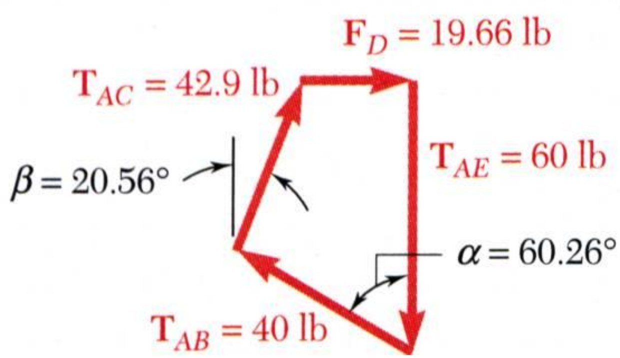
$$\tan\alpha = \frac{7\text{ ft}}{4\text{ ft}} = 1.75 \qquad \tan\beta = \frac{1.5\text{ ft}}{4\text{ ft}} = 0.375$$

$$\alpha = 60.25° \qquad \beta = 20.56°$$

- Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.

$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AE} + \vec{F}_D = 0$$

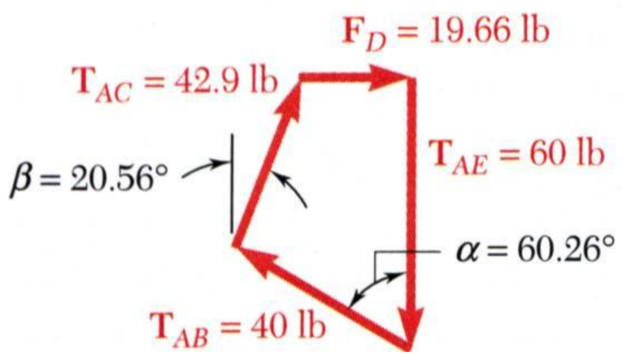




- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.

$$\vec{T}_{AB} = -(40\,\text{lb})\sin 60.26°\,\vec{i} + (40\,\text{lb})\cos 60.26°\,\vec{j}$$
$$= -(34.73\,\text{lb})\vec{i} + (19.84\,\text{lb})\vec{j}$$
$$\vec{T}_{AC} = T_{AC}\sin 20.56°\,\vec{i} + T_{AC}\cos 20.56°\,\vec{j}$$
$$= 0.3512\,T_{AC}\,\vec{i} + 0.9363\,T_{AC}\,\vec{j}$$
$$\vec{T} = -(60\,\text{lb})\vec{i}$$
$$\vec{F}_D = F_D\,\vec{i}$$

$$\vec{R} = 0$$
$$= (-34.73 + 0.3512\,T_{AC} + F_D)\,\vec{i}$$
$$+ (19.84 + 0.9363\,T_{AC} - 60)\,\vec{j}$$



$$\vec{R} = 0$$
$$= \left(-34.73 + 0.3512 T_{AC} + F_D\right)\vec{i}$$
$$+ \left(19.84 + 0.9363 T_{AC} - 60\right)\vec{j}$$

This equation is satisfied only if each component of the resultant is equal to zero

$$\left(\sum F_x = 0\right) \quad 0 = -34.73 + 0.3512 T_{AC} + F_D$$
$$\left(\sum F_y = 0\right) \quad 0 = 19.84 + 0.9363 T_{AC} - 60$$

$$\boxed{\begin{aligned} T_{AC} &= +42.9 \text{ lb} \\ F_D &= +19.66 \text{ lb} \end{aligned}}$$