Axial Loading

Concept of Stress & Strain

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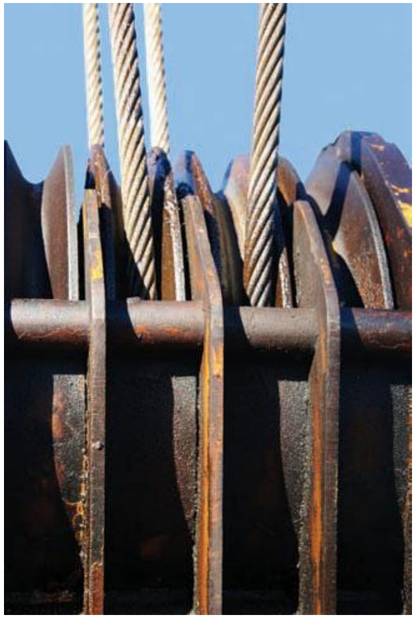
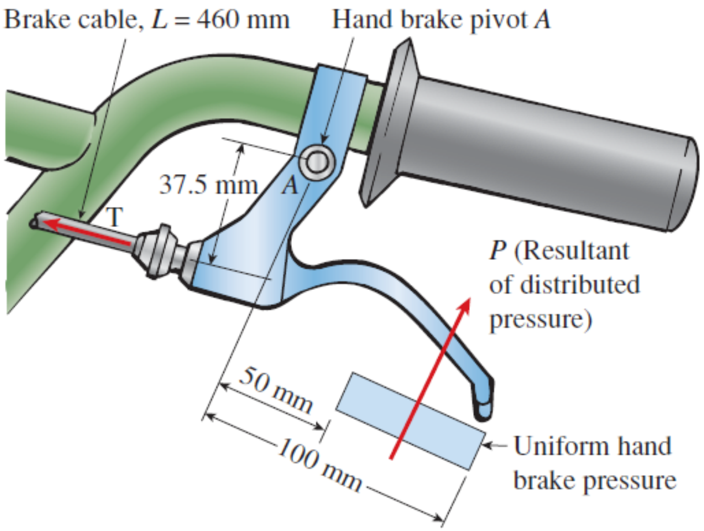
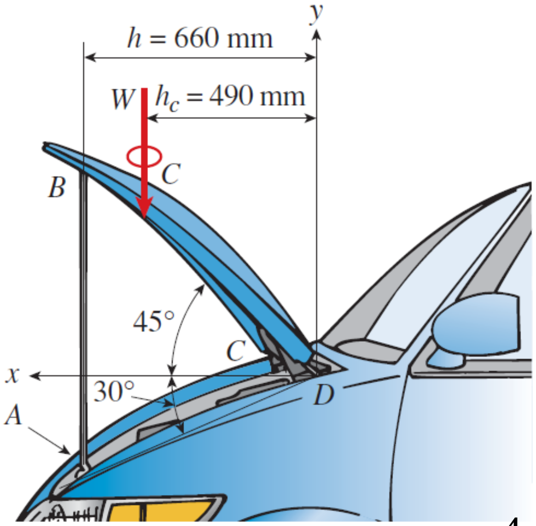
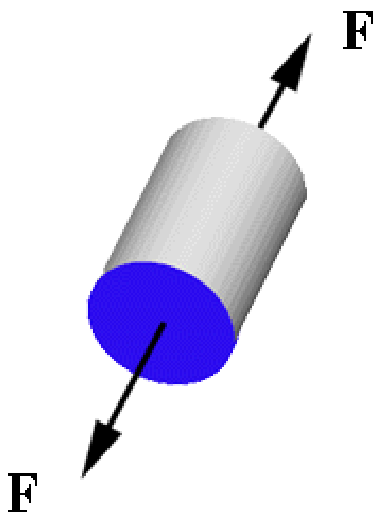
Introduction

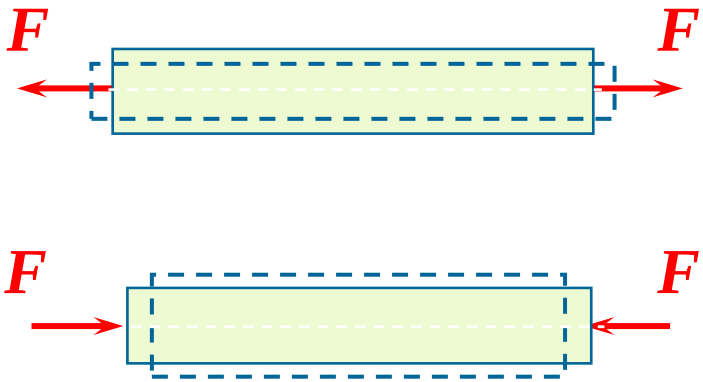





Introduction





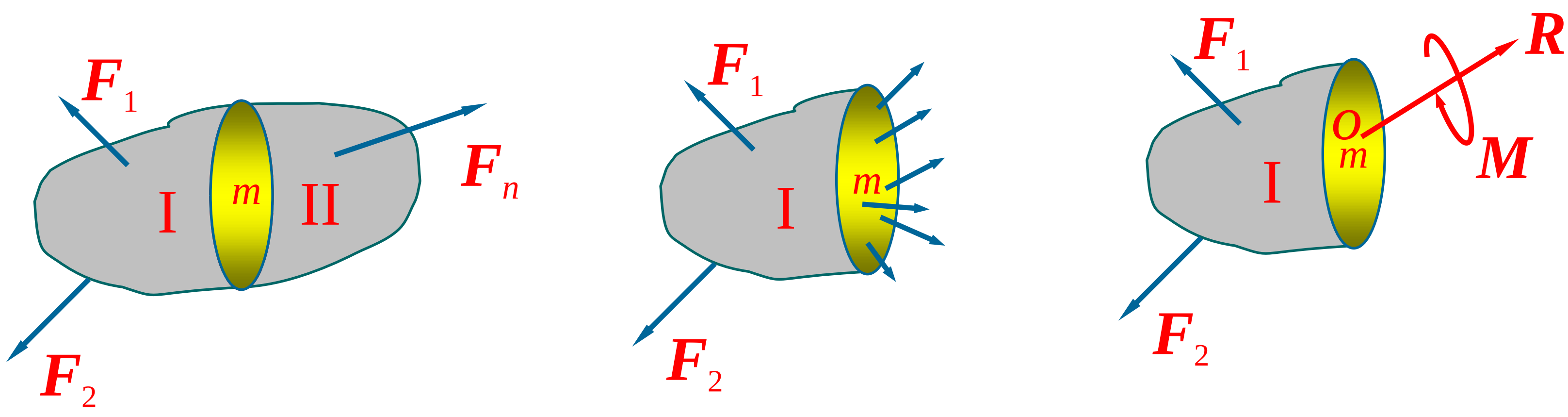
- **Load:** equal and opposite forces along bar axis
- **Deformation:** extension (contraction) along bar axis and contraction (extension) transversely

Internal Forces Illustrated by Method of Sections

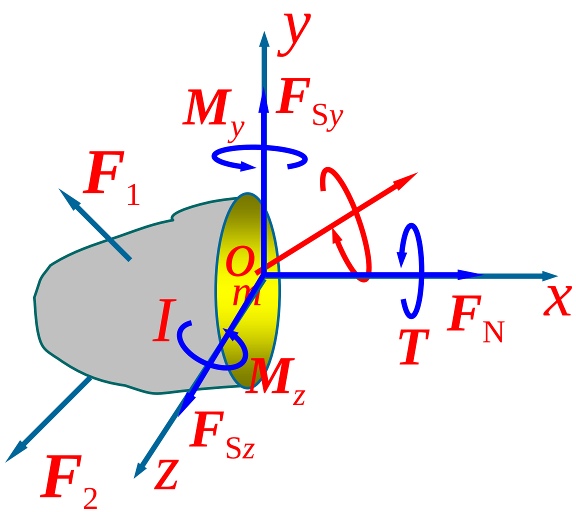
Internal Force

- The change of interaction force among various parts of a solid body, introduced by external loading.

The Method of Section



Sign Convention of Axial Forces

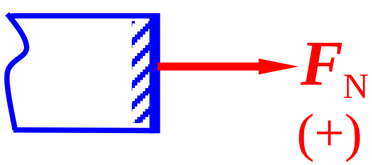


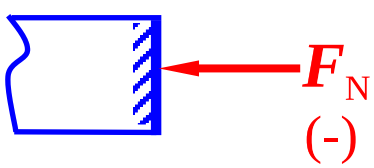
- **Coordinate:** a right-handed system

  O: Centroid of the cross-section

  x: Cross-section normal (Bar axis)

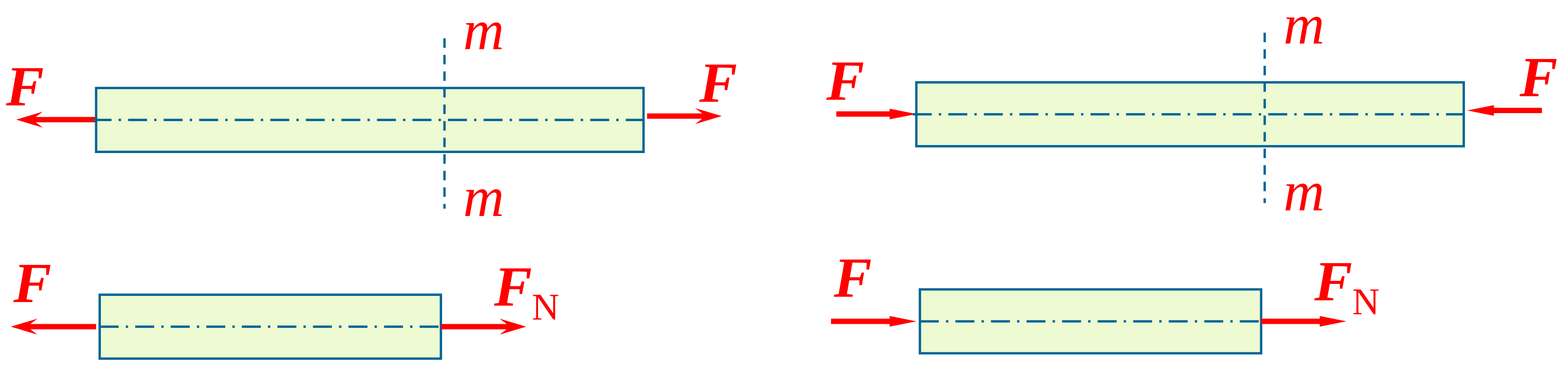
- **Forces:** F_N, F_{Sy}, F_{Sz}
- **Moments:** T, M_y, M_z
- **Axial Force (F_N)**
  - Along bar axis
  - Extends or contracts the bar along bar axis
  - Positive for tension; Negative for compression
  - Assuming positive for all unknown axial forces





Procedure of Method of Sections

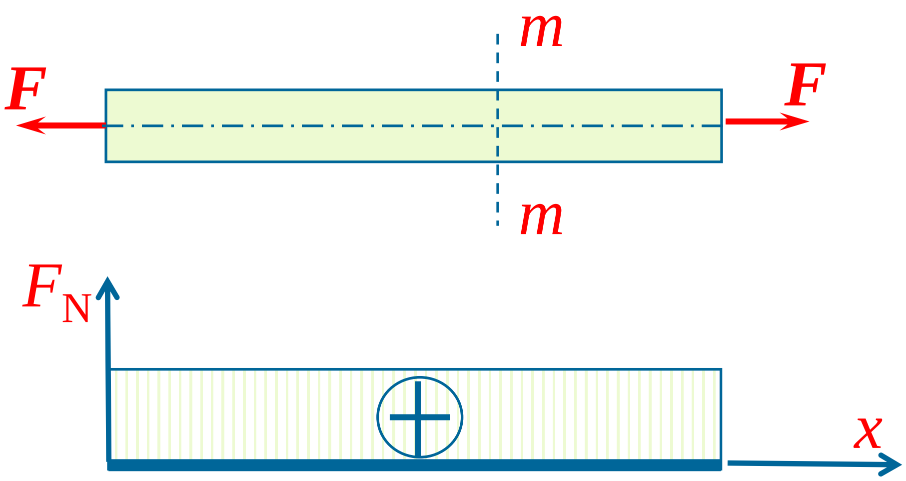
- Sectioning the member
- Taking either portion
- Substituting the other portion with internal forces
- Equilibrating



$F_N = F$ $F_N = -F$

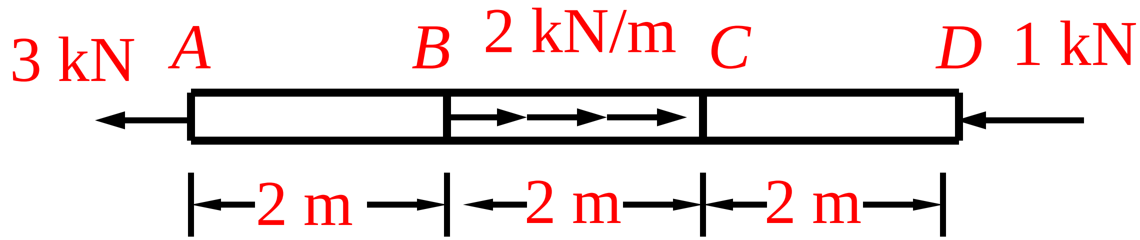
Diagram of Axial Forces

- Abscissa: position of cross sections
- Ordinate: axial force
- Positive for tension; negative for compression.



Sample Problem

- Plot the diagram of axial force

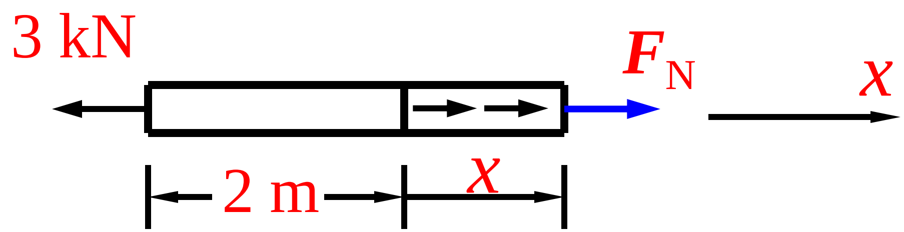


- Solution

1. Internal force in *AB*、*CD*

$$F_{NAB} = 3\ \text{kN}$$

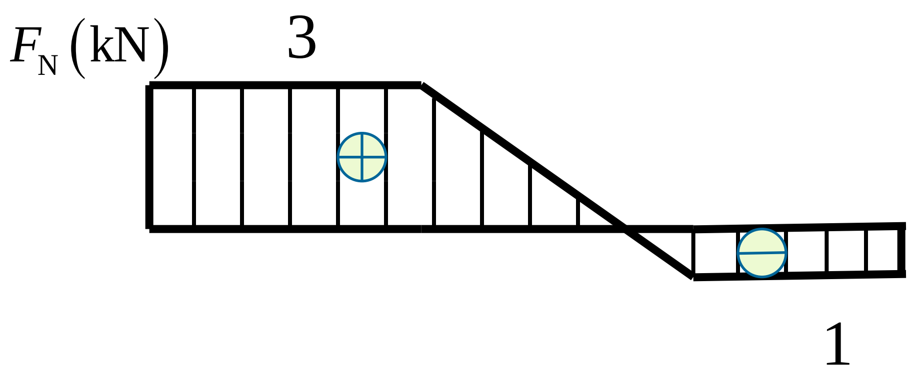
$$F_{NCD} = -1\ \text{kN}$$



2. Internal force in BC

$$F_N(x) = 3 - 2x \quad (0 < x < 2)$$

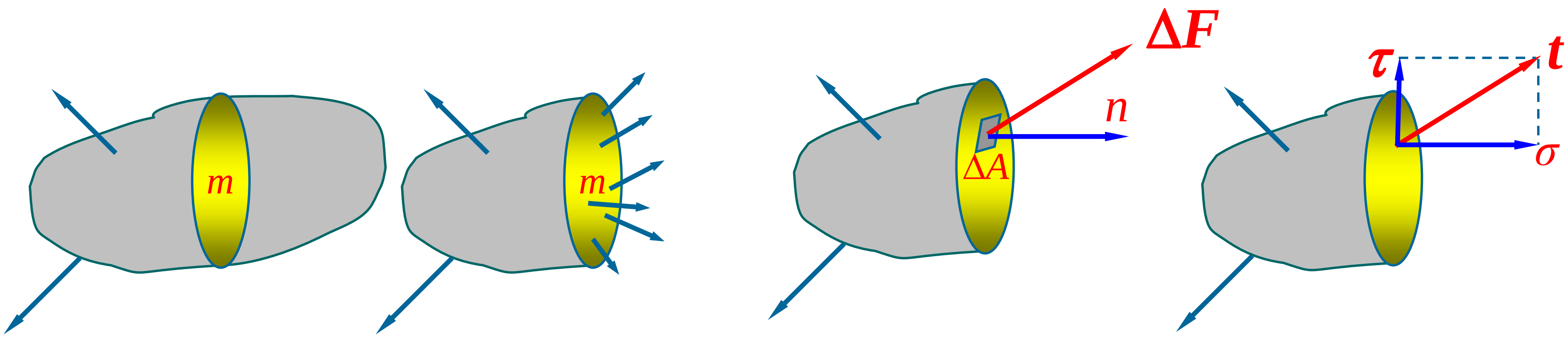
3. Diagram of axial force



4. Maximum internal force

$$\left|F_{N,\max}\right| = 3\ \text{kN}$$

Concept of Stresses

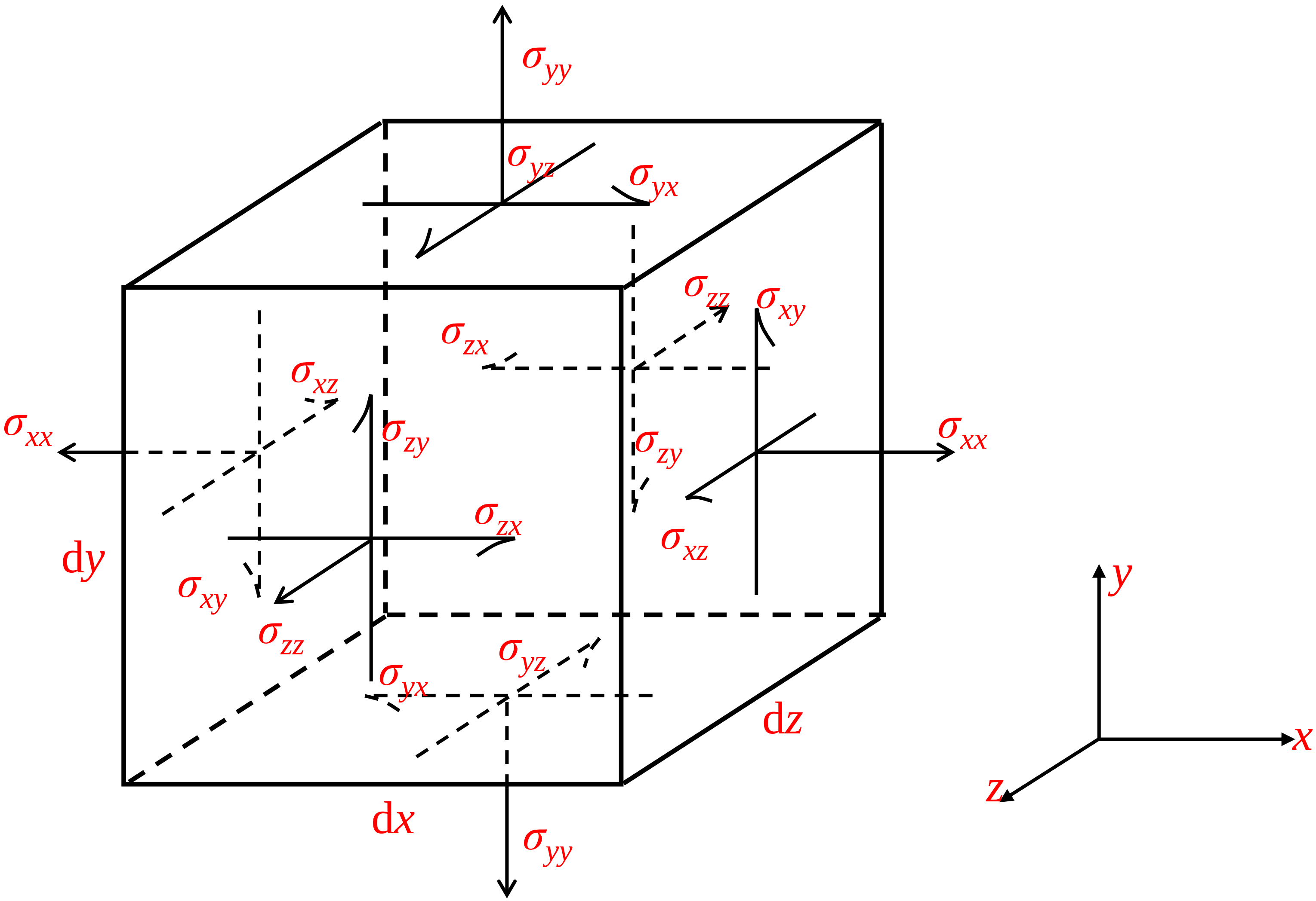


Given ΔF as the force transmitted across ΔA, a stress traction vector can be defined as

$$t = \lim_{\Delta A \to 0} \Delta F / \Delta A$$

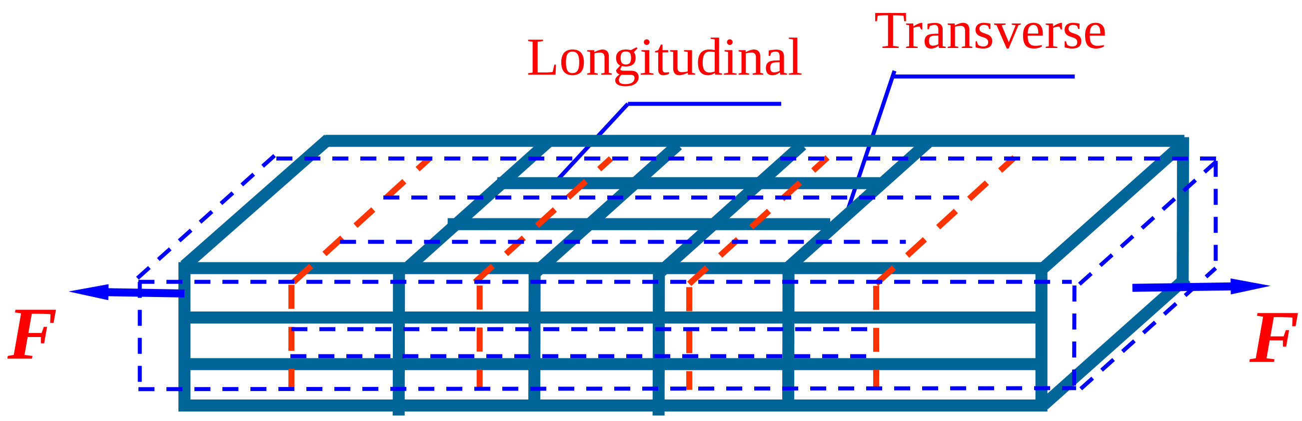
Units: Pa (N/m^2), 1 MPa = 10^6 Pa, 1 GPa = 10^9 Pa.

General Stress State of a Point

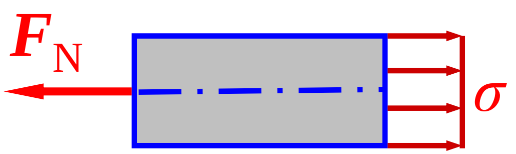


All stress components shown in the above figure are positive.

Stresses Acting on Cross Sections



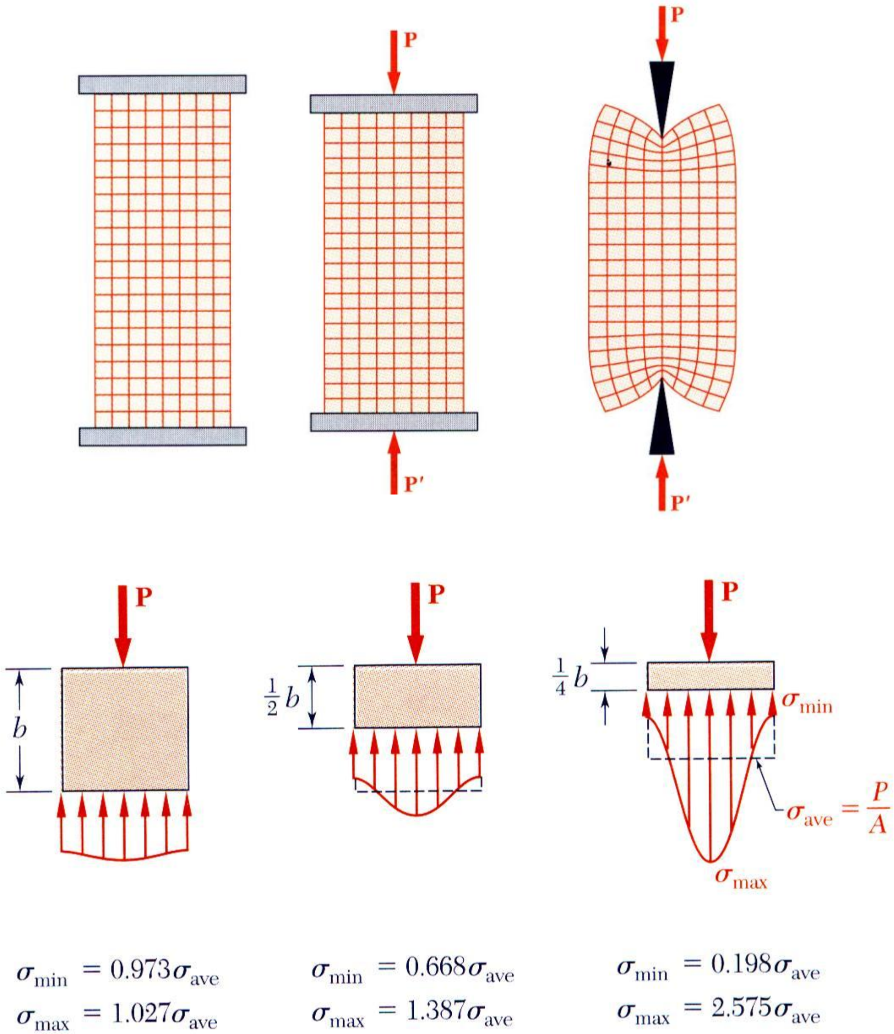
Assumption: stresses are uniformly distributed on the cross sections of axially loaded bars (Saint-Venant's Principle)



$$F_N = \int_A \sigma \mathrm{d}A = \sigma \int_A \mathrm{d}A = \sigma A$$

$$\sigma = \frac{F_N}{A}$$

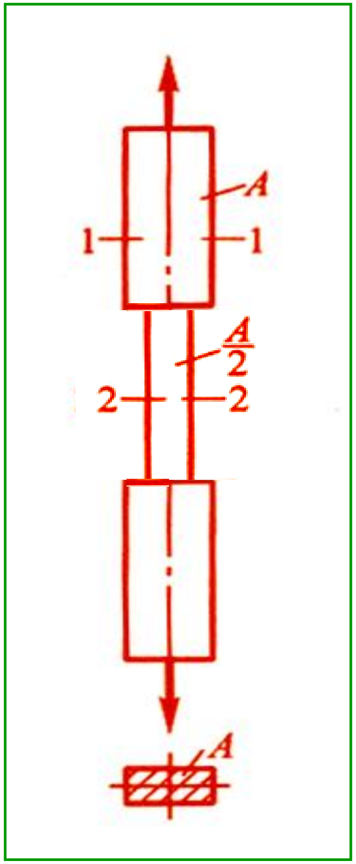
Saint-Venant's Principle



- Loads transmitted through rigid plates result in uniform distribution of stress and strain.
- Concentrated loads result in large stresses in the vicinity of the load application point.
- Stress and strain distributions become uniform at a relatively short distance from the load application points.
- **Saint-Venant's Principle:** Stress distribution may be assumed independent of the mode of load application except in the immediate vicinity of load application points.

Sample Problem

$$\sigma = \frac{F_{\mathrm{N}}}{A}$$

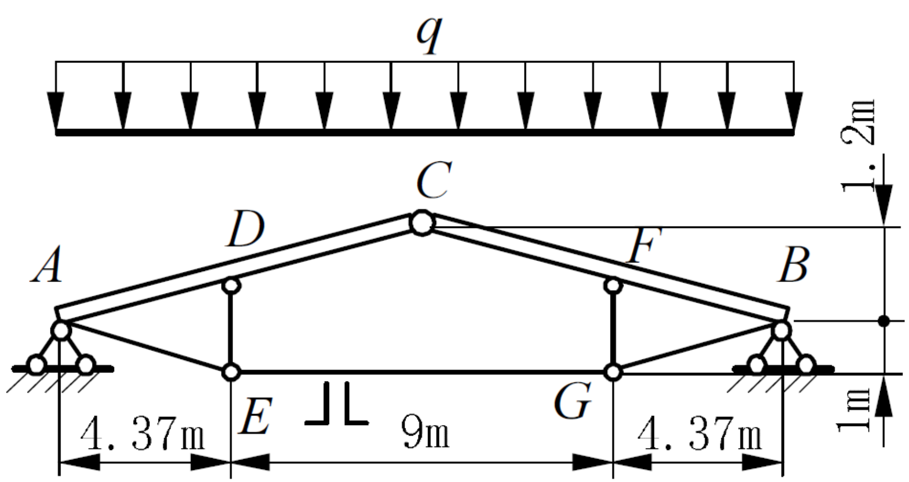


$$A_1 = 2A_2$$

$$\sigma_{A2} = 2\sigma_{A1}$$

Sample Problem

- For the compound roof structure shown, determine the axial stress developed in members *AE* and *EG*. q = 20 kN/m, $A_{AE} = A_{EG}$ = 23 cm^2.

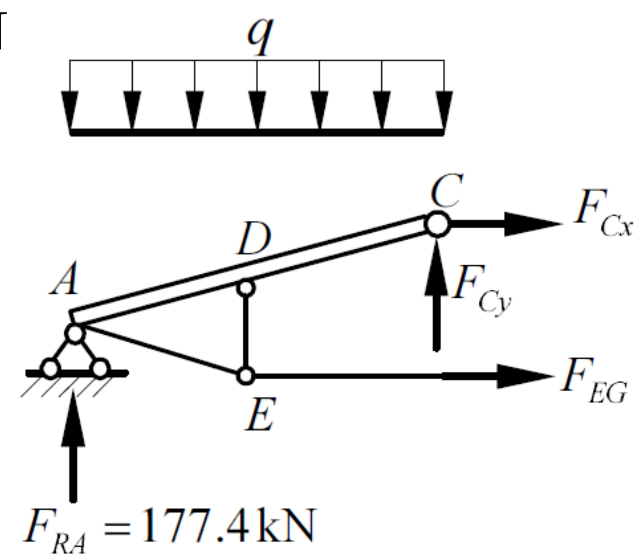


- Solution

1. Reaction force

$$0=\sum F_y \Rightarrow F_{RA}=F_{RB}=\frac{ql}{2}=177.4\text{ kN}$$

2. Axial force in *EG*



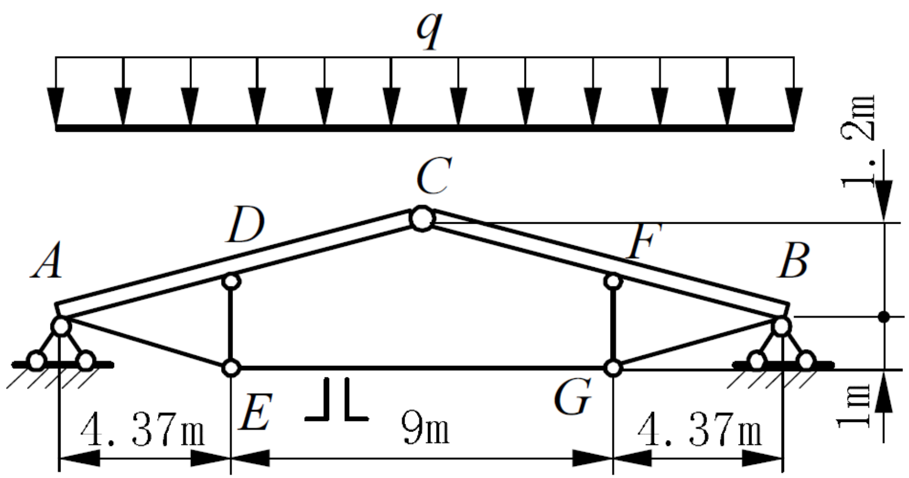
$$0=\sum M_C=\frac{q}{2}(4.37+4.5)^2$$
$$-F_{RA}(4.37+4.5)+F_{EG}(1.2+1)$$
$$\Rightarrow F_{EG}=356\text{ kN}$$

3. Equilibrium at joint *E*

$$F_{EG} = F_{AE}\cos\alpha$$

$$= F_{AE}\frac{4.37}{\sqrt{4.37^2+1^2}}$$

$$\Rightarrow F_{AE} = 366\ \text{kN}$$

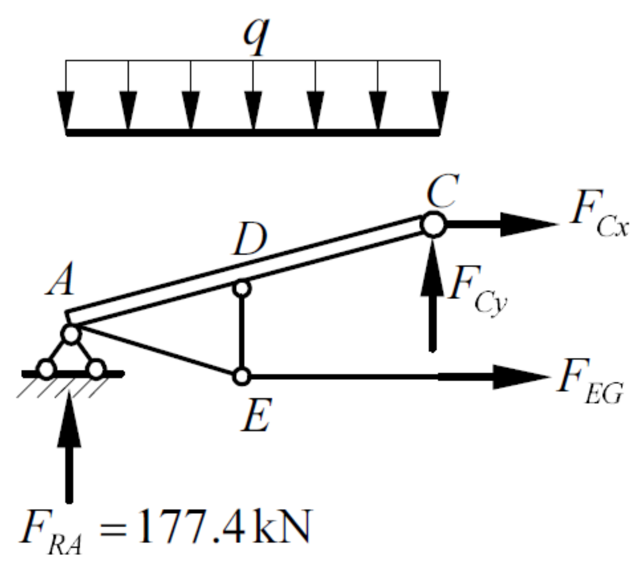


4. Axial stresses in members

AE and *EG*

$$\sigma_{EG} = \frac{F_{EG}}{A_{EG}} = \frac{356(10^3)}{23(10^{-4})} = 155\ \text{MPa}$$

$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}} = \frac{366(10^3)}{23(10^{-4})} = 159\ \text{MPa}$$



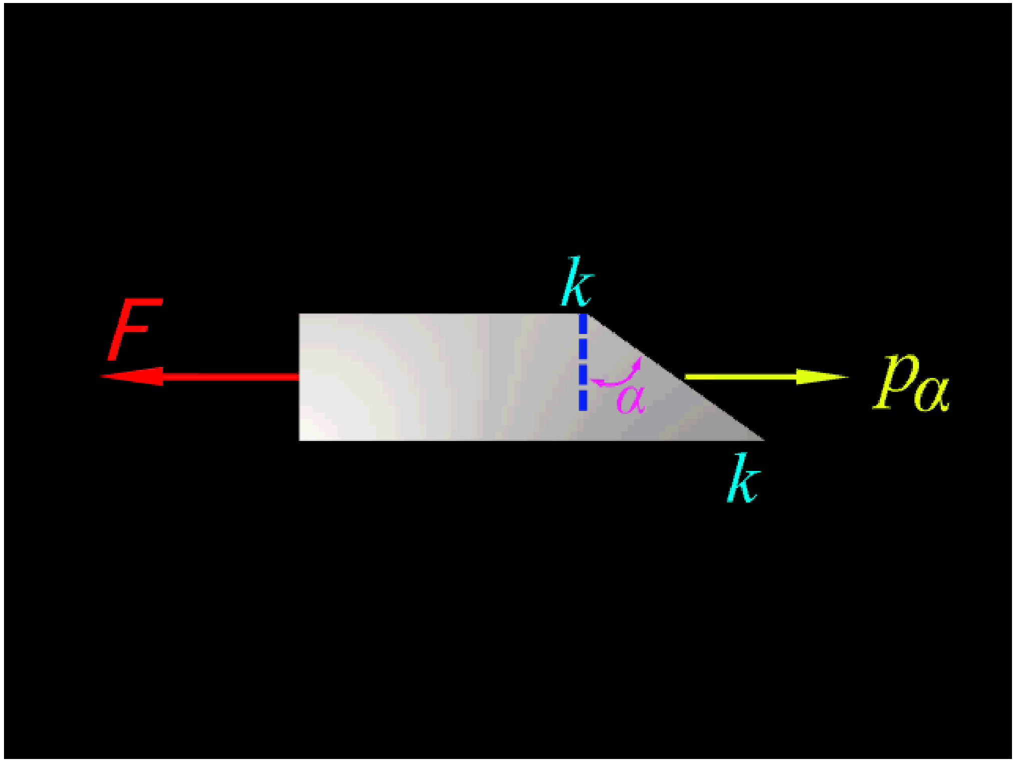
Stresses Acting on Oblique Sections

$$A_\alpha = A / \cos\alpha$$

$$p_\alpha = \frac{F}{A_\alpha} = \frac{F}{A}\cos\alpha = \sigma\cos\alpha$$

$$\sigma_\alpha = p_\alpha \cos\alpha = \sigma\cos^2\alpha$$

$$\tau_\alpha = p_\alpha \sin\alpha = \frac{\sigma}{2}\sin 2\alpha$$

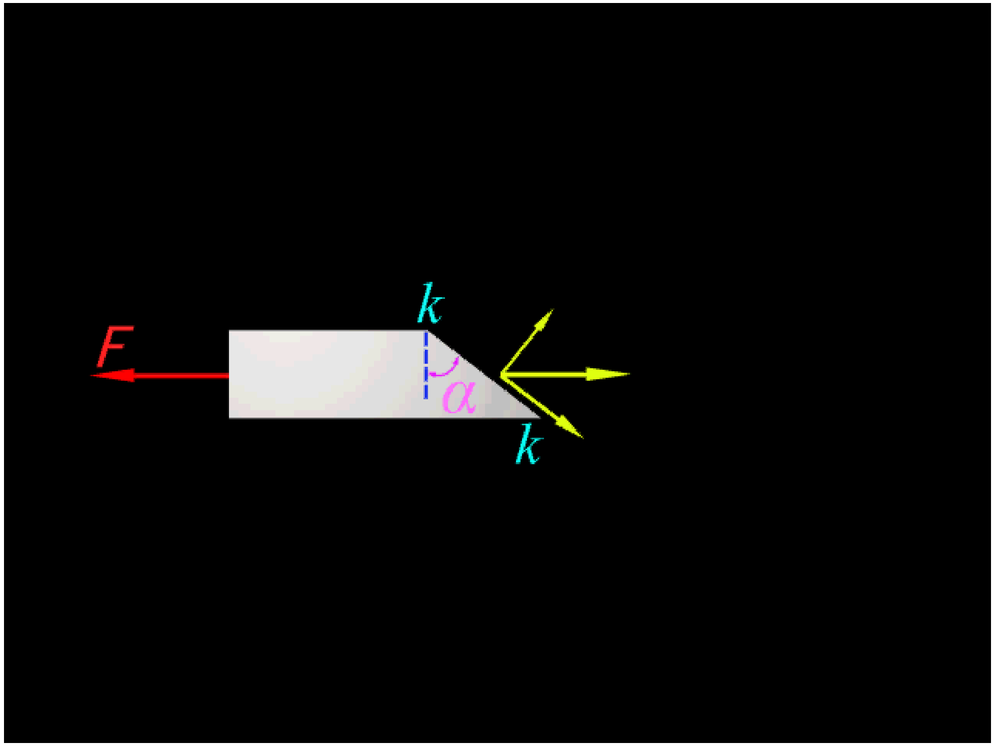


***α*:** defined as the angle measured from cross section normal to oblique cross section normal.

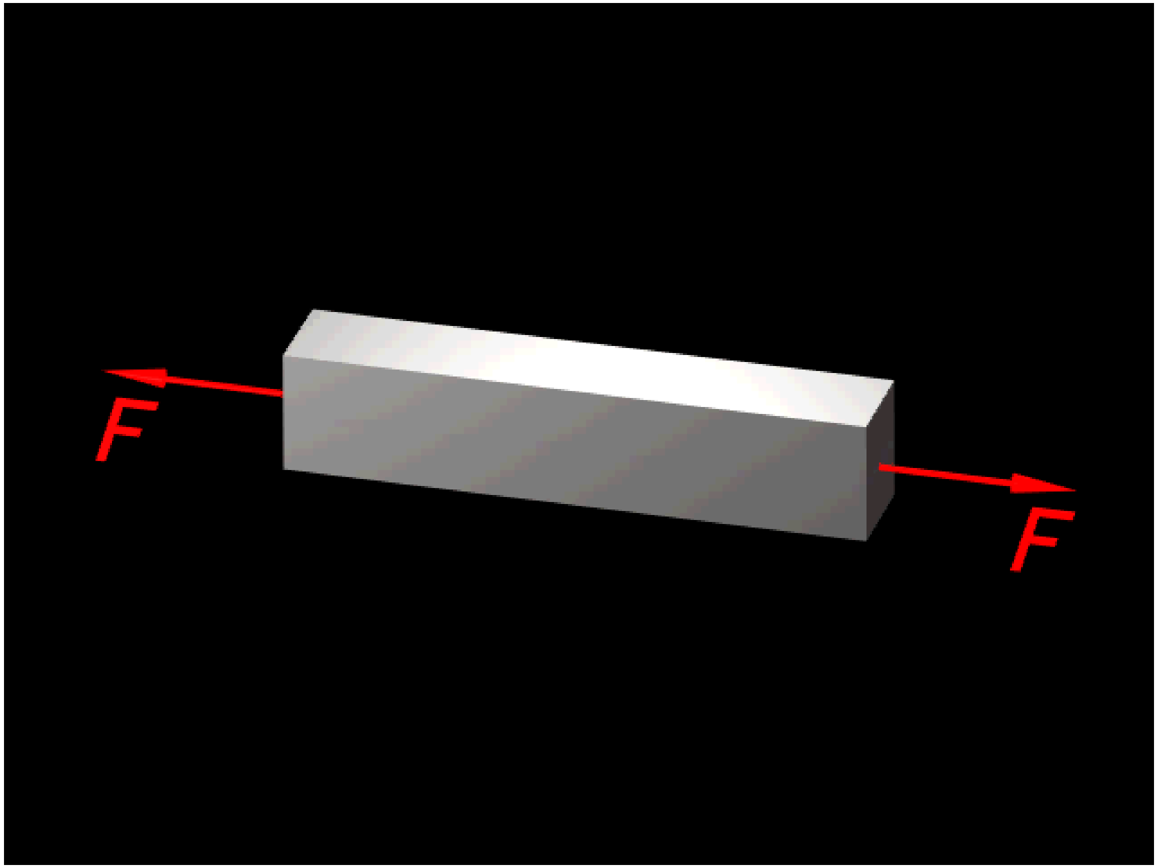
- On cross section: maximum normal stress; zero shearing stress.

- On 45°oblique cross section: maximum shearing stress.

$$\tau_{max} = \tau_{45^0} = \frac{\sigma}{2}\sin(90^0) = \frac{\sigma}{2} \quad , \quad \sigma_{45^0} = \sigma\cos^2(45^0) = \frac{\sigma}{2}$$

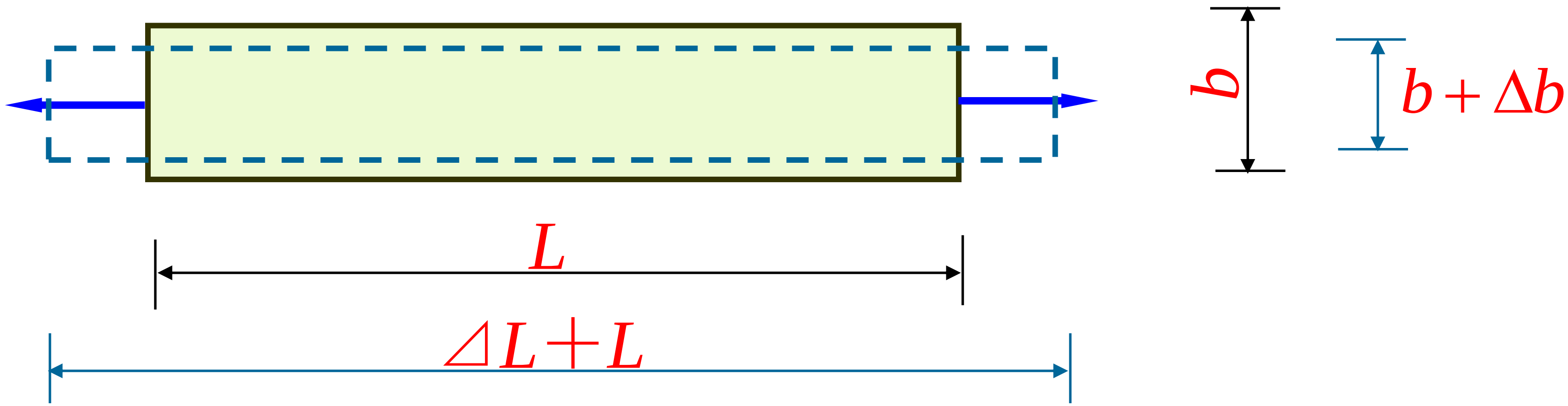


- On 90°oblique cross section: zero normal and shearing stress.



Deformation of Axially Loaded Bars

- Longitudinal (axial) & transverse strains



- Longitudinal strain:

$$\varepsilon = \Delta L / L$$

- Transverse strain:

$$\varepsilon' = \Delta b / b$$

- Connection between longitudinal and transverse strain

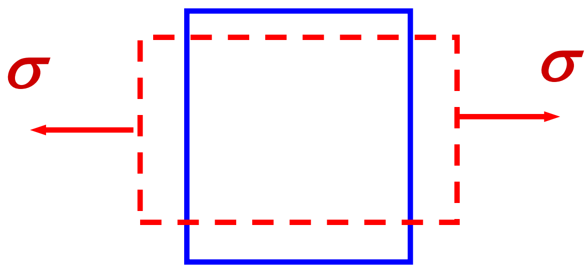
$$\varepsilon' = -\nu\varepsilon \qquad \nu = -\varepsilon' / \varepsilon$$

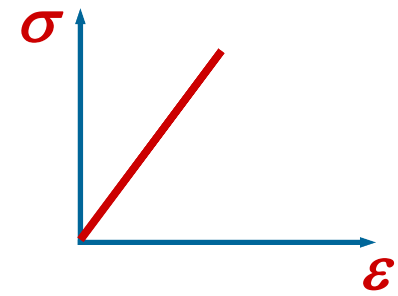
- ν: Poisson's ratio

Deformation of Axially Loaded Bars

- Consider a differential cube of side length *a*, *b* and *c*

$$F = \sigma(bc) = K\Delta L = Ka\varepsilon \quad \Rightarrow \sigma = (Ka/bc)\varepsilon \equiv E\varepsilon$$





- This linear relation is referred to as the one-dimensional Hooke’s law, with *E* denoting the Young’s modulus.

- The measurement of axial deformation

$$\Delta L = L\varepsilon = L\frac{\sigma}{E} = \frac{LF_N}{AE} = \frac{F_N L}{EA}$$

- *EA*: Tension (Compression) rigidity

Elastic Constants of Engineering Materials

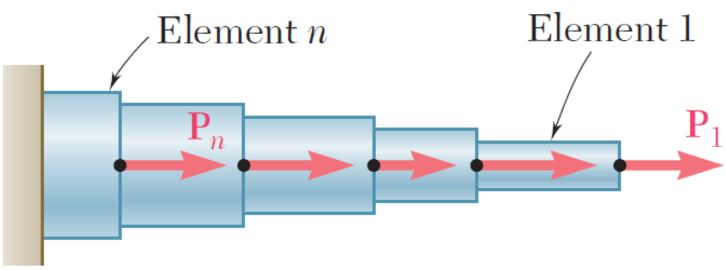
Material	Modulus of elasticity E		Shear modulus of elasticity G		Poisson's ratio ν
	ksi	GPa	ksi	GPa	
Aluminum alloys	10,000–11,400	70–79	3,800–4,300	26–30	0.33
2014-T6	10,600	73	4,000	28	0.33
6061-T6	10,000	70	3,800	26	0.33
7075-T6	10,400	72	3,900	27	0.33
Brass	14,000–16,000	96–110	5,200–6,000	36–41	0.34
Bronze	14,000–17,000	96–120	5,200–6,300	36–44	0.34
Cast iron	12,000–25,000	83–170	4,600–10,000	32–69	0.2–0.3
Concrete (compression)	2,500–4,500	17–31			0.1–0.2
Copper and copper alloys	16,000–18,000	110–120	5,800–6,800	40–47	0.33–0.36
Glass	7,000–12,000	48–83	2,700–5,100	19–35	0.17–0.27
Magnesium alloys	6,000–6,500	41–45	2,200–2,400	15–17	0.35
Monel (67% Ni, 30% Cu)	25,000	170	9,500	66	0.32

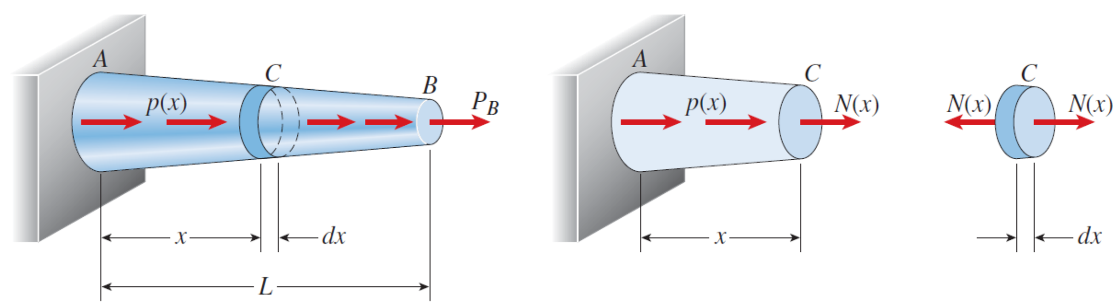
Elastic Constants of Engineering Materials

Material	Modulus of elasticity E		Shear modulus of elasticity G		Poisson's ratio ν
	ksi	GPa	ksi	GPa	
Nickel	30,000	210	11,400	80	0.31
Plastics					
Nylon	300–500	2.1–3.4			0.4
Polyethylene	100–200	0.7–1.4			0.4
Rock (compression)					
Granite, marble, quartz	6,000–14,000	40–100			0.2–0.3
Limestone, sandstone	3,000–10,000	20–70			0.2–0.3
Rubber	0.1–0.6	0.0007–0.004	0.03–0.2	0.0002–0.001	0.45–0.50
Steel	28,000–30,000	190–210	10,800–11,800	75–80	0.27–0.30
Titanium alloys	15,000–17,000	100–120	5,600–6,400	39–44	0.33
Tungsten	50,000–55,000	340–380	21,000–23,000	140–160	0.2
Wood (bending)					
Douglas fir	1,600–1,900	11–13			
Oak	1,600–1,800	11–12			
Southern pine	1,600–2,000	11–14			

Nonuniform Tension/compression

- Pure tension/compression formula refers to a prismatic bar subjected to axial forces acting only at the ends.
- Nonuniform tension/compression differs from pure tension in that the bar need not to be prismatic and the applied axial forces may act anywhere along the axis of the bar.
- Bars in nonuniform tension/compression can be analyzed by applying the formulas of pure tension/compression to finite segments of the bar and then adding the results, or by applying the formulas to differential elements of the bar and then integrating.



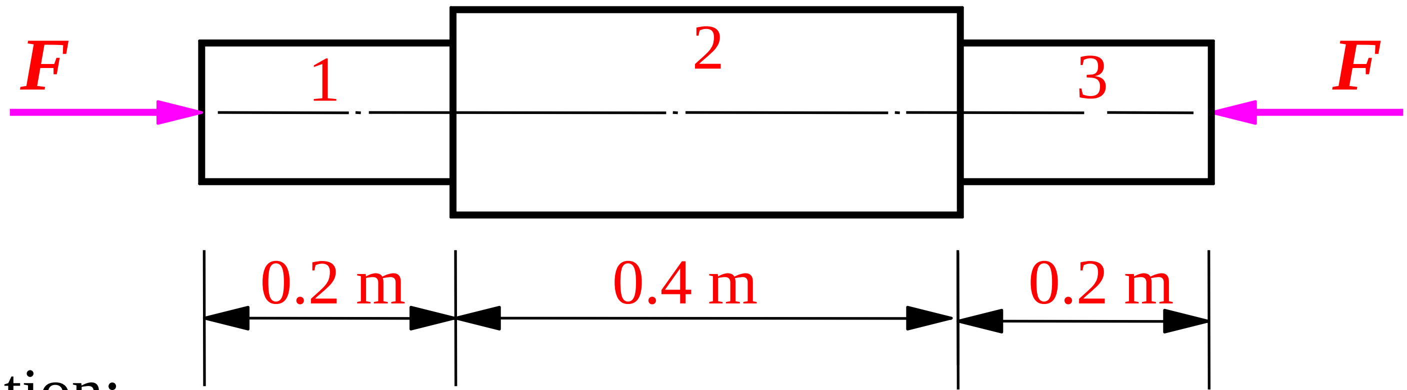


$$\Delta L = \sum_i \Delta L_i = \sum_i \frac{F_{Ni} L_i}{E_i A_i}$$

$$\Delta L = \int_0^L d\Delta L = \int_0^L \frac{F_N(x)\,dx}{EA(x)}$$

Sample Problem

- Consider an axially loaded bar with varied cross section. Given: E = 210 GPa; Section 1 (circular): d_1 = 20 mm; Section 2 (square): side length a = 25 mm, σ_2 = -30 MPa; Section 3 (circular) d_3=12mm. Find: Total change in bar length $\triangle L$.

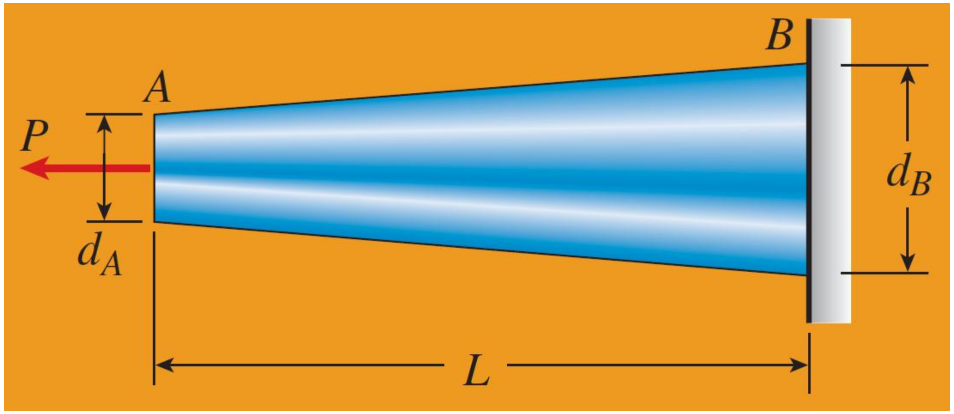


- Solution:

$$F = \sigma_2 A_2 = -30\ \text{MPa} \times 25^2\ \text{mm}^2 = -18.75\ \text{kN}$$

$$\Delta L = \frac{F_{N_1} L_1}{EA_1} + \frac{F_{N_2} L_2}{EA_2} + \frac{F_{N_3} L_3}{EA_3} = \frac{-18750}{210 \times 10^9} \left(\frac{0.2}{\frac{\pi \times 0.02^2}{4}} + \frac{0.4}{0.025^2} + \frac{0.2}{\frac{\pi \times 0.012^2}{4}} \right) = -0.272\text{mm}$$

Sample Problem



- Determine the elongation of the bar due to the end load *P*.

- Solution:

$$d_x = d_A + \frac{x}{L}\left(d_B - d_A\right)$$

$$A(x) = \frac{\pi d_x^2}{4} = \frac{\pi}{4}\left(d_A + \frac{x}{L}\left(d_B - d_A\right)\right)^2$$

$$\Delta_L = \int_0^L \frac{F_N dx}{EA(x)} = \int_0^L \frac{4P dx}{E\pi\left(d_A + \frac{x}{L}\left(d_B - d_A\right)\right)^2}$$

$$= \frac{4PL}{\pi E\left(d_B - d_A\right)} \int_0^L \frac{d\left(d_A + \frac{x}{L}\left(d_B - d_A\right)\right)}{\left(d_A + \frac{x}{L}\left(d_B - d_A\right)\right)^2}$$

$$= \frac{4PL}{\pi E\left(d_B - d_A\right)}\left[-\frac{1}{d_A + \frac{x}{L}\left(d_B - d_A\right)}\right]_0^L$$

$$= \frac{4PL}{\pi E\left(d_B - d_A\right)}\left[\frac{1}{d_A} - \frac{1}{d_B}\right] = \frac{4PL}{\pi E d_A d_B}$$

- Cannot be assumed as a prismatic bar that has the diameter $(d_A + d_B)/2$.
- For the special case of a prismatic bar:

$$\Delta_L = \frac{4PL}{\pi E d^2} = \frac{PL}{EA}$$

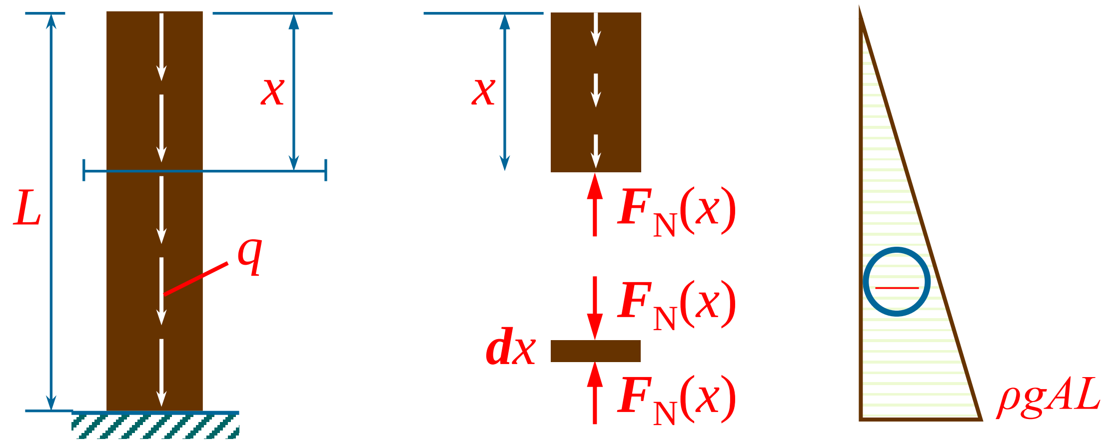
Sample Problem

- Deformation of a uniform bar due to gravity. Given: cross section area A, density ρ, Young's modulus E. Find: Maximum normal stress and axial length change.
- Solution:

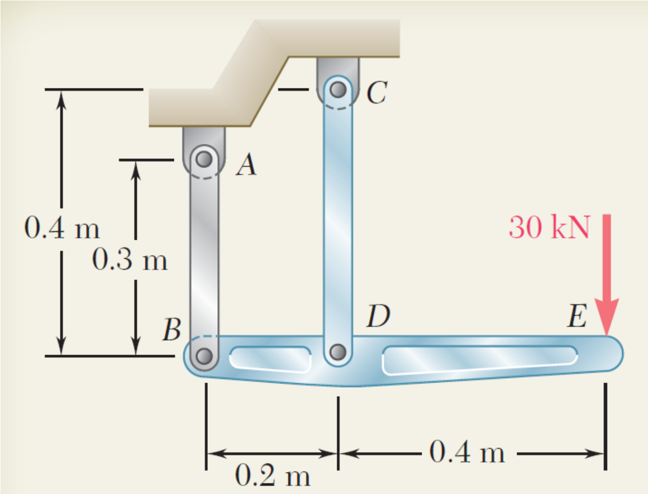
1. Axial forces and stresses

$$q=\rho gA;\ \ F_N(x)=\rho gAx;\ \ \sigma(x)=F_N(x)/A=\rho gx,\ \ \sigma_{max}=\rho gL$$

2. Deformation: $\mathbf{d}(\Delta L)=\dfrac{F_N(x)\mathrm{d}x}{EA} \Rightarrow \Delta L=\displaystyle\int_0^l \frac{F_N(x)\mathrm{d}x}{EA}=\int_0^l \frac{\rho gAx\mathrm{d}x}{EA}=\frac{\rho gL^2}{2E}$



Sample Problem

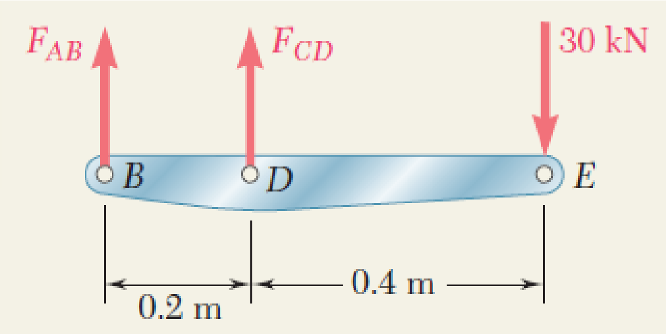


- The rigid bar *BDE* is supported by two links *AB* and *CD*.
- Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2. Link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of (600 mm^2).
- For the 30-kN force shown, determine the deflection a) of *B*, b) of *D*, and c) of *E*.

SOLUTION:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at *E* given the deflections at *B* and *D*.

- SOLUTION:
- Free body: Bar *BDE*



$$\sum M_B = 0$$

$$0 = -(30\,\text{kN} \times 0.6\,\text{m}) + F_{CD} \times 0.2\,\text{m}$$

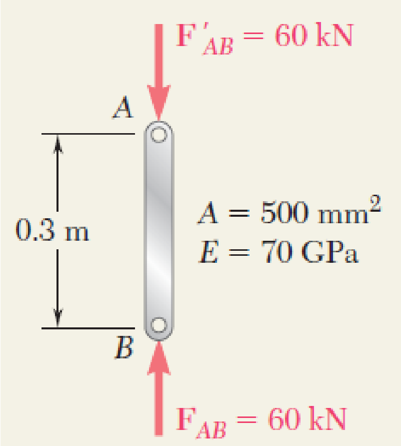
$$F_{CD} = +90\,\text{kN} \quad \textit{tension}$$

$$\sum M_D = 0$$

$$0 = -(30\,\text{kN} \times 0.4\,\text{m}) - F_{AB} \times 0.2\,\text{m}$$

$$F_{AB} = -60\,\text{kN} \quad \textit{compression}$$

- Displacement of *B*:



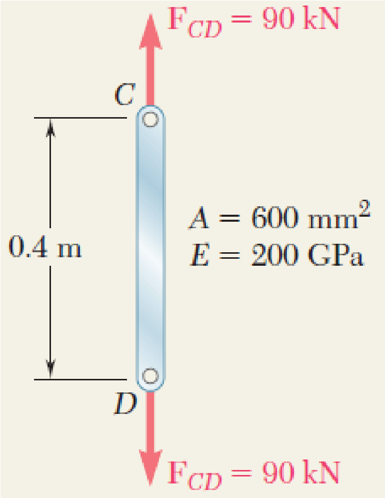
$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60\times10^3\,\text{N})(0.3\,\text{m})}{(500\times10^{-6}\,\text{m}^2)(70\times10^9\,\text{Pa})}$$

$$= -514\times10^{-6}\,\text{m}$$

$$\boxed{\delta_B = 0.514\,\text{mm} \uparrow}$$

- Displacement of *D*:

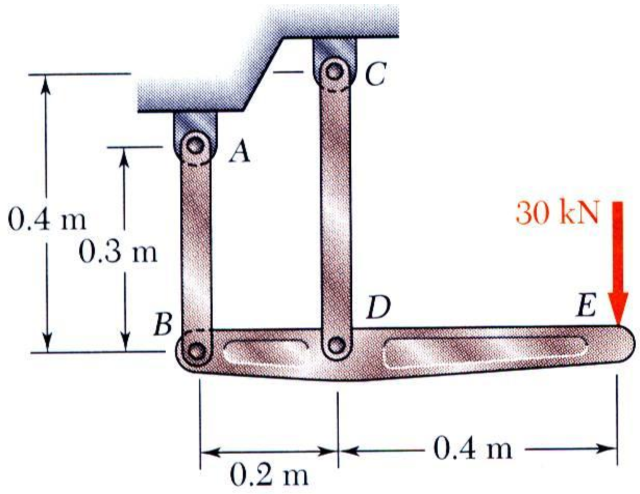


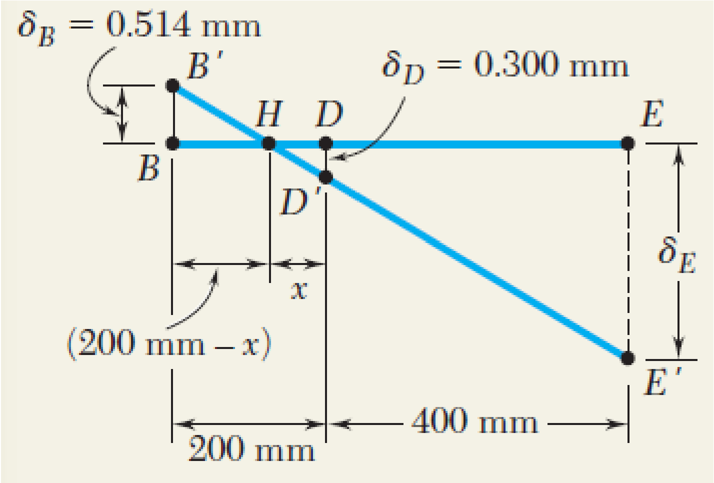
$$\delta_D = \frac{PL}{AE}$$

$$= \frac{(90\times10^3\,\text{N})(0.4\,\text{m})}{(600\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{Pa})}$$

$$= 300\times10^{-6}\,\text{m}$$

$$\boxed{\delta_D = 0.300\,\text{mm} \downarrow}$$





- Displacement of *E*:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514\,\text{mm}}{0.300\,\text{mm}} = \frac{(200\,\text{mm}) - x}{x}$$

$$x = 73.7\text{ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

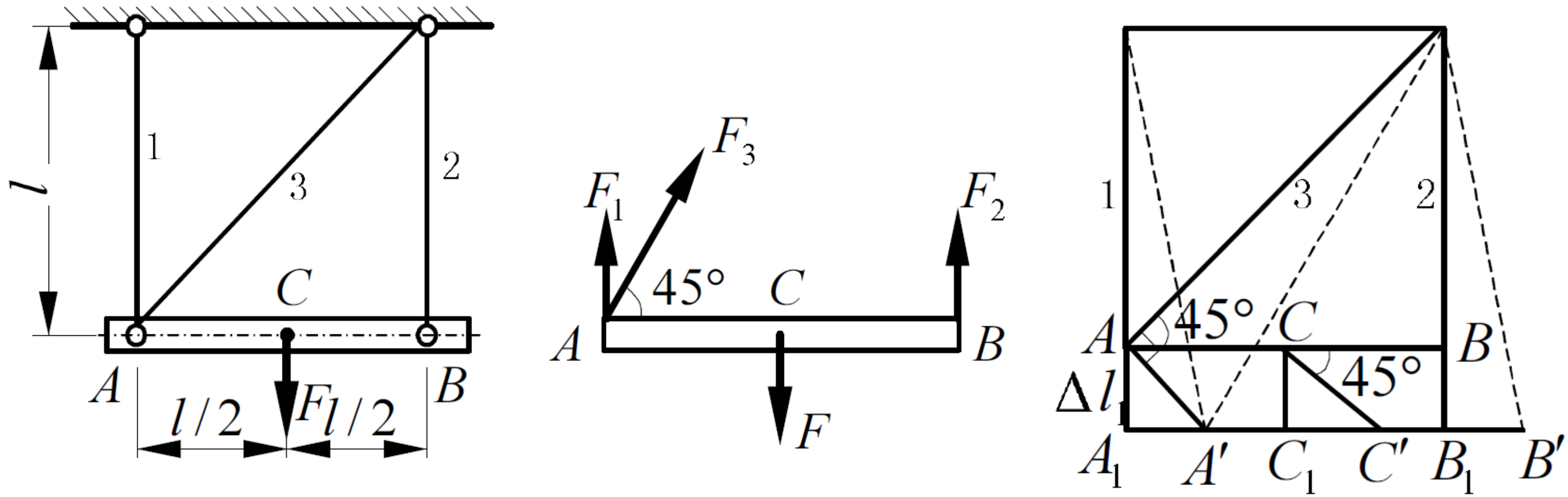
$$\frac{\delta_E}{0.300\,\text{mm}} = \frac{(400 + 73.7)\text{mm}}{73.7\text{ mm}}$$

$$\delta_E = 1.928\,\text{mm}$$

$$\boxed{\delta_E = 1.928\,\text{mm} \downarrow}$$

Sample Problem

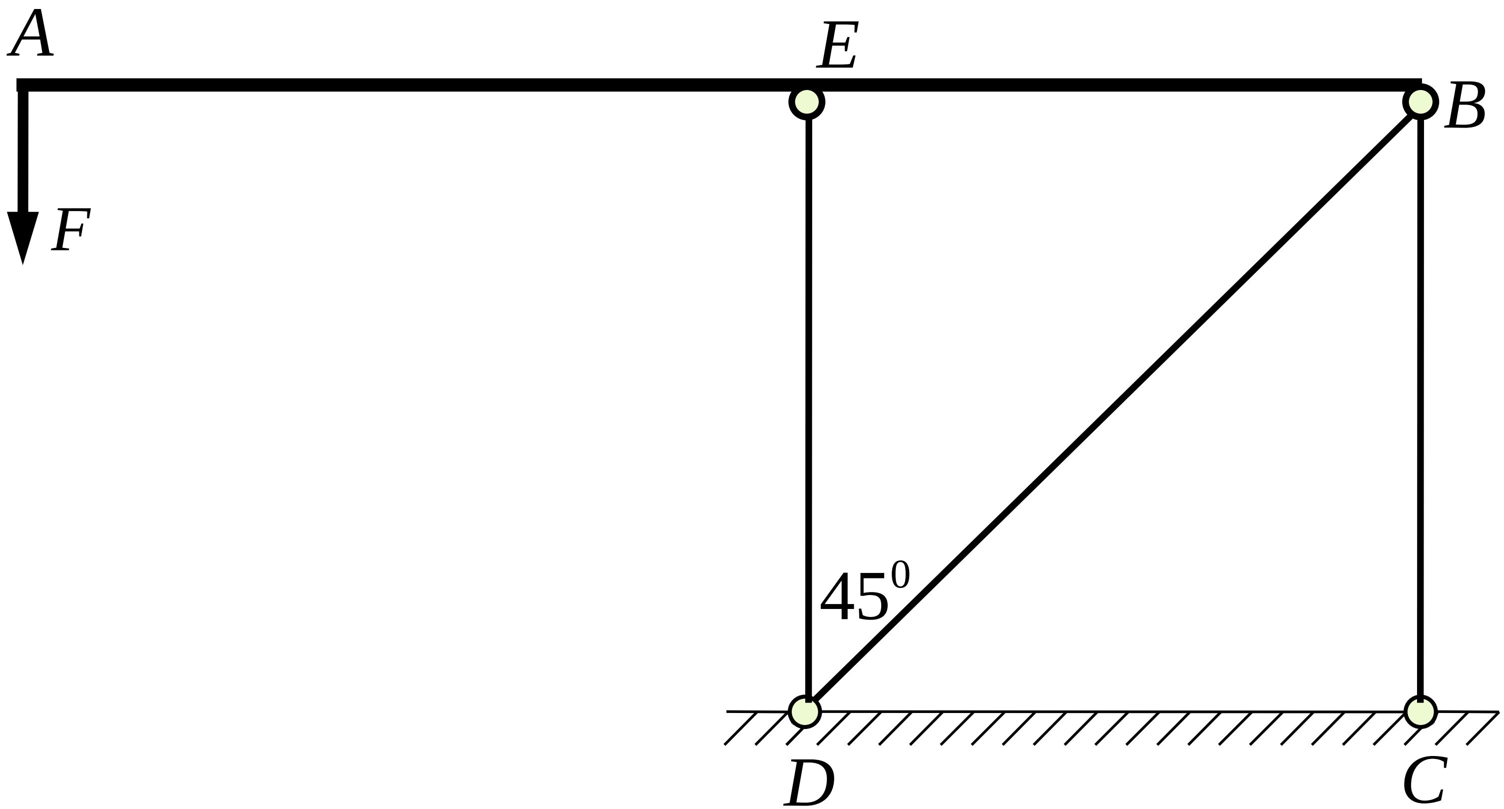
- Given: tension rigidity of bar 1, 2 and 3 = EA, AB rigid, F and L. Find: the vertical and horizontal displacement at C.



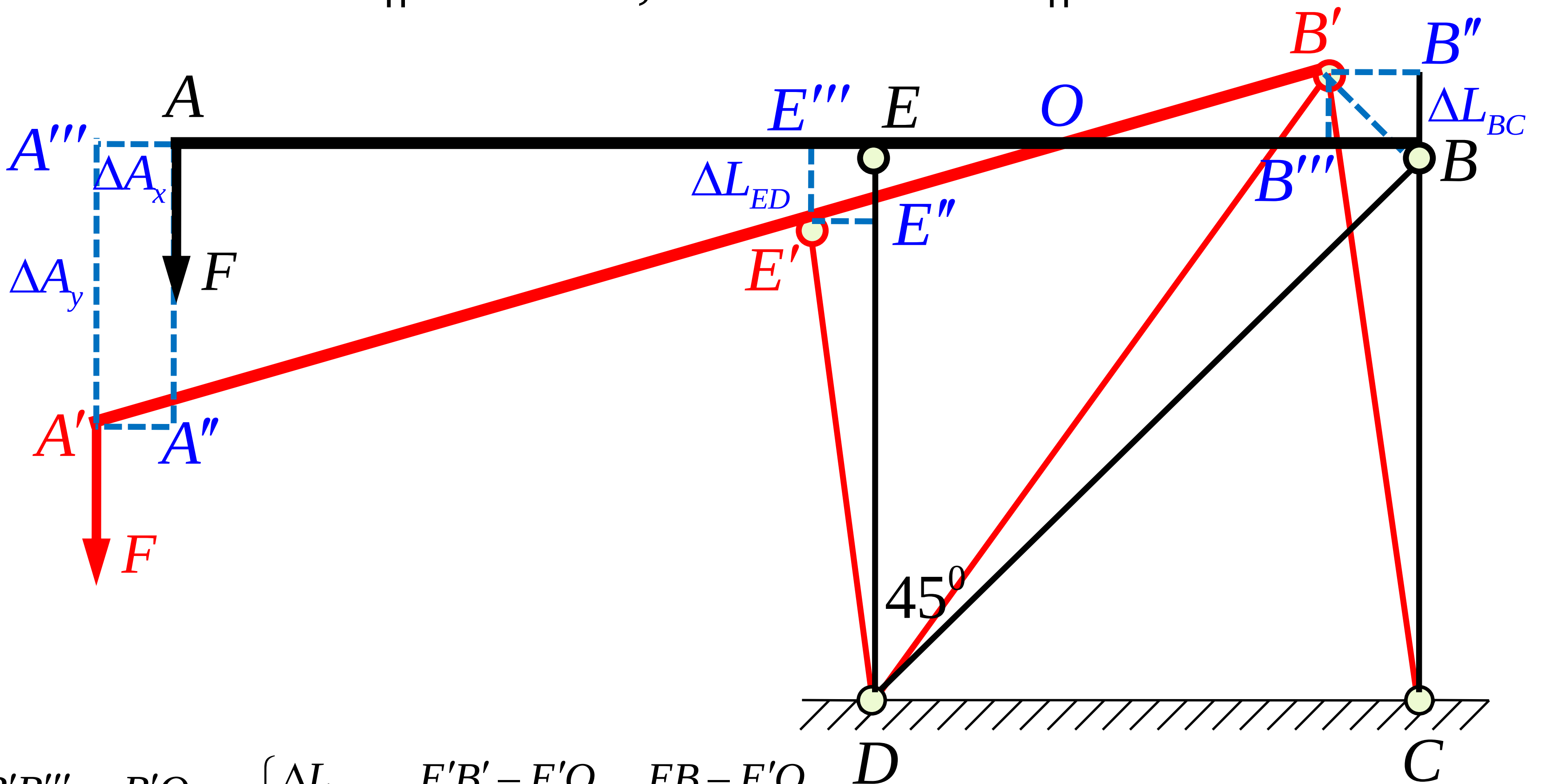
$$\Delta x_C = \Delta y_C = \frac{FL}{2EA}$$

Sample Problem

- Determine the vertical and horizontal displacement of *A*. *AB* is rigid.

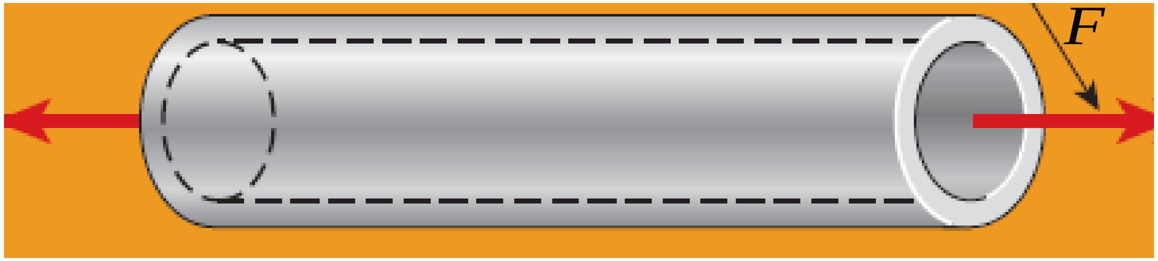


- Hint: $\Delta B'B'''O \parallel \Delta E'E'''O,\qquad \Delta A'A'''O \parallel \Delta E'E'''O$



Sample Problem (Poisson's Ratio)

- A circular tube of inner diameter *d* = 60 mm and outer diameter *D* = 80 mm is subjected to an axial load of *F* = 200 kN, determine the wall thickness under the deformed state. *E* = 200 GPa, ν = 0.3.



- Solution:

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{EA} = \frac{200\left(10^{3}\right)}{200\left(10^{9}\right)\pi\left(80^{2}-60^{2}\right)\left(10^{-6}\right)/4} = 455\left(10^{-6}\right)$$

$$\varepsilon' = -\nu\varepsilon = -0.3(455)\left(10^{-6}\right) = -136.5\left(10^{-6}\right)$$

$$\delta' = \frac{(D-d)}{2}(1+\varepsilon') = 10\left(1-136.5\left(10^{-6}\right)\right) = 9.998635 \text{ mm}$$

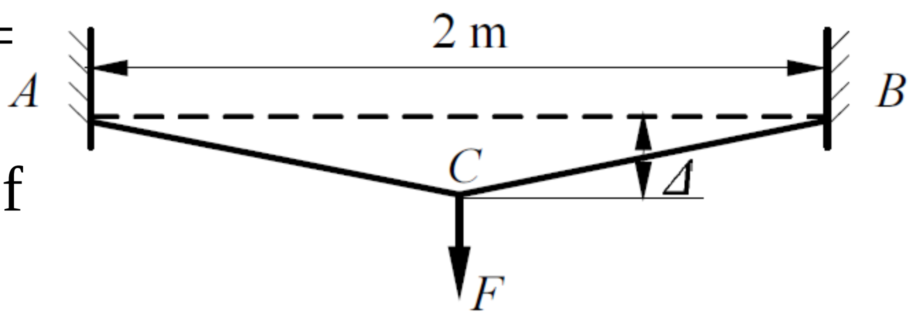
$$S' = \pi D(1+\varepsilon') = (251.327412)\left(1-136.5\left(10^{-6}\right)\right) = 251.293106 \text{ mm}$$

$$s' = \pi d(1+\varepsilon') = (188.495559)\left(1-136.5\left(10^{-6}\right)\right) = 188.469830 \text{ mm}$$

- Poisson's ratio is the same for all transverse directions. (Isotropy)

Sample Problem (Large Deformation)

- Given: d = 1 mm, ε = 0.0035, E = 210 GPa. Determine the axial stress in wire, the displacement of C, and the force F.



- Solution:

1. Axial stress in the wire: $\sigma = E\varepsilon = 210\left(10^9\right)\left(0.0035\right) = 735\ \text{MPa}$

2. The displacement of C

$$\Delta L_{AC} = L_{AC}\varepsilon = (1)(0.0035) = 0.0035\ \text{m}$$

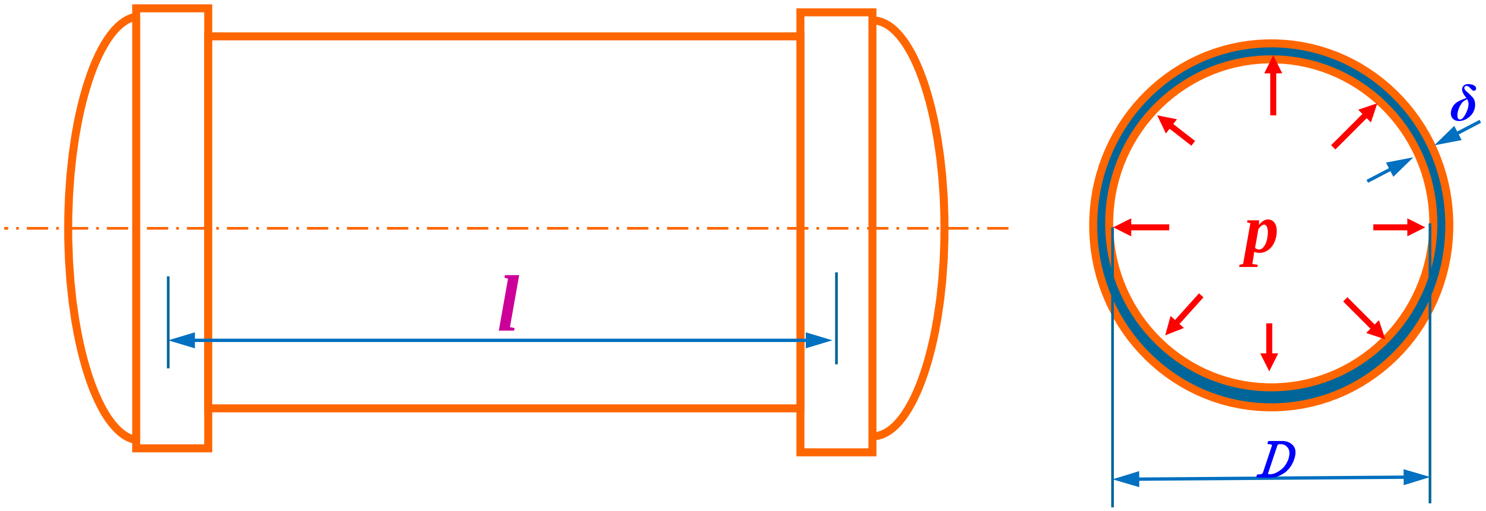
$$\Delta_C = \sqrt{\left(L_{AC} + \Delta L_{AC}\right)^2 - L_{AC}^2} = \sqrt{2L_{AC}\Delta L_{AC} + \Delta L_{AC}^2} = \sqrt{2(1)(0.0035) + 0.0035^2} = 0.0837\ \text{m}$$

3. The force F (Equilibrium must be analyzed under the deformed state.)

$$F = 2F_{\text{N}} \sin\angle BAC = 2(\sigma A)\frac{\Delta_C}{L'_{AC}} = 2(735)\left(10^6\right)\left(\pi\left(10^{-3}\right)^2/4\right)\frac{0.0837}{1.0035} = 96.3\ \text{N}$$

Sample Problem (Pressure Vessel)

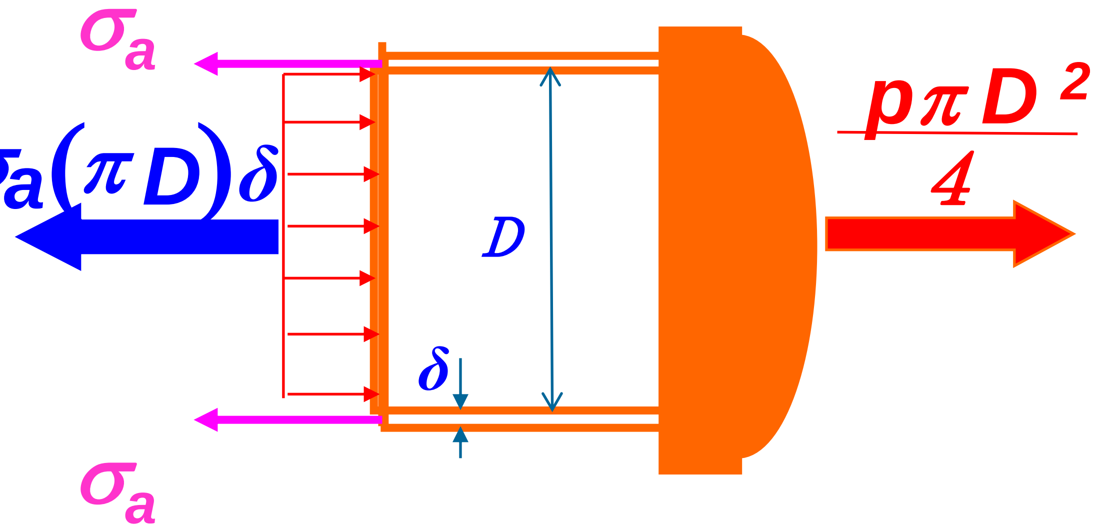
- For the thin-walled pressure vessel shown, D, δ, and p denote the vessel inner diameter, wall thickness and pressure respectively. Determine: the axial stress, circumferential stress, and the elongation of vessel circumference, diameter and length.



$$\sum F_{\text{axial}} = 0$$

$$\Rightarrow \sigma_{\text{a}} \left(\pi D \delta\right) = p\left(\pi D^2 / 4\right)$$

$$\Rightarrow \sigma_{\text{a}} = \frac{pD}{4\delta}$$

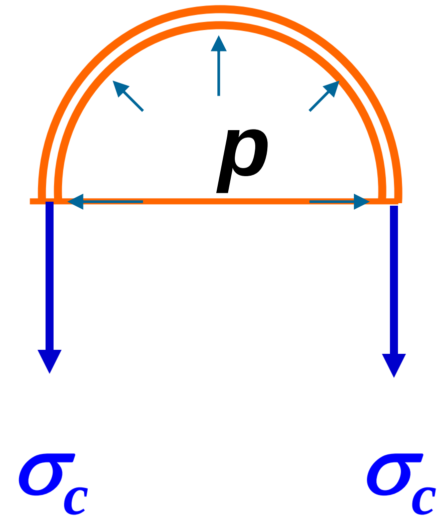


$$\sum F_{\text{circumferential}} = 0$$

$$\Rightarrow \sigma_c \left(2\delta \times b\right) = \int_0^{\pi} \sin\theta \, p b \frac{D}{2} d\theta$$

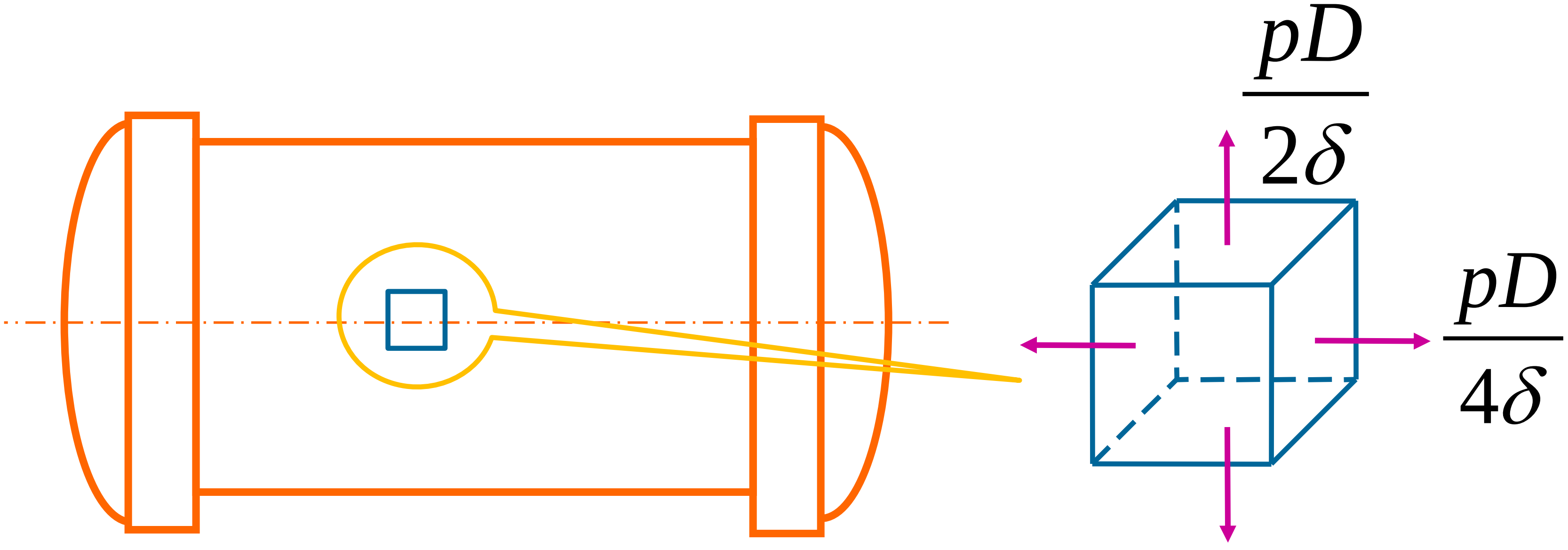
$$\Rightarrow 2\delta b \sigma_c = pbD$$

$$\Rightarrow \sigma_c = \frac{pD}{2\delta}$$



- *b*: an arbitrarily chosen axial length

- Circumferential stress: $\sigma_c = pD/2\delta$
- Axial stress: $\sigma_a = pD/4\delta$
- Radial stress: $\sigma_D = -p$



- Elongation of the vessel circumference

$$\Delta_c = \frac{F_{\mathrm{N}} L}{EA} = \frac{\left(\sigma_t \cancel{A}\right)\left(\pi\left(D+\delta\right)\right)}{E\cancel{A}} = \frac{pD}{2\delta}\frac{\pi\left(D+\delta\right)}{E} = \frac{\pi pD^2}{2E\delta}\left(1+\frac{\delta}{D}\right) \approx \frac{\pi pD^2}{2E\delta}$$

- Elongation of the vessel diameter

$$\Delta_D = \frac{\Delta_t}{\pi} = \frac{pD^2}{2E\delta}\left(1+\frac{\delta}{D}\right) \approx \frac{pD^2}{2E\delta}$$

- Elongation of the vessel length

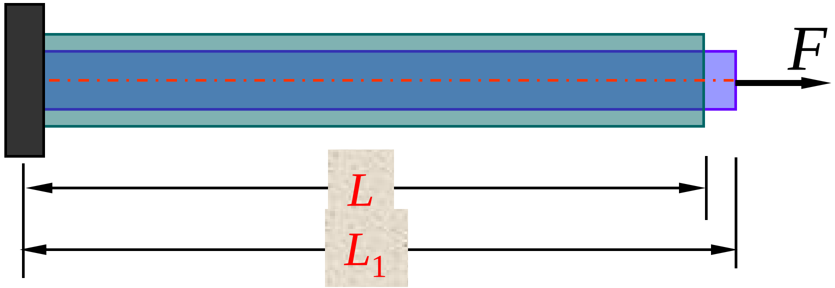
$$\Delta_a = \frac{F_{\mathrm{N}} L}{EA} = \frac{\left(\sigma_a \cancel{A}\right) l}{E\cancel{A}} = \frac{pDl}{4E\delta}$$

- Transverse strain

$$\varepsilon_c = \frac{\Delta_t}{L_t} = \frac{pD}{2E\delta} \qquad = \varepsilon_D = \frac{\Delta_D}{L_D} = \frac{pD}{2E\delta}$$

Strain Energy

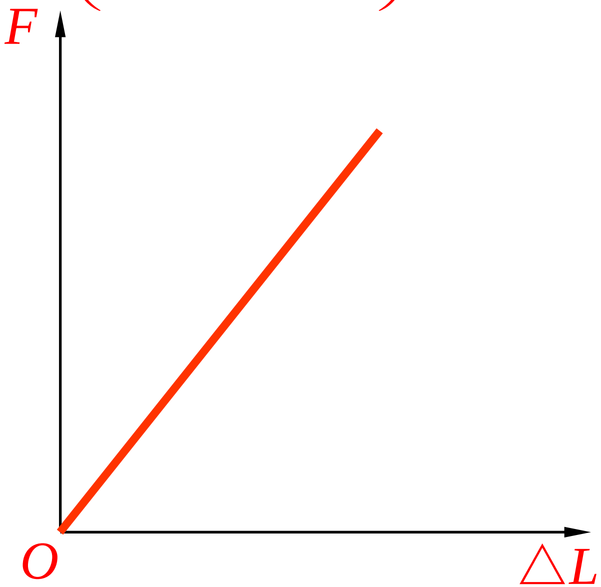
- Strain energy is developed in solid materials due to elastic deformation induced by external loading.



- Strain energy and work in pure tension/compression

$$U = W = \frac{1}{2} F_{\mathrm{N}} \Delta L = \frac{F_{\mathrm{N}}^2 L}{2EA} = \frac{EA(\Delta L)^2}{2L}$$

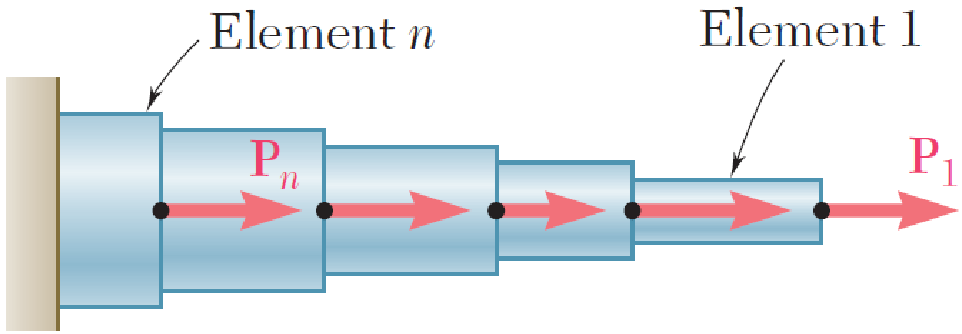
$$\left(\Delta L = \frac{F_{\mathrm{N}} L}{EA} \right)$$



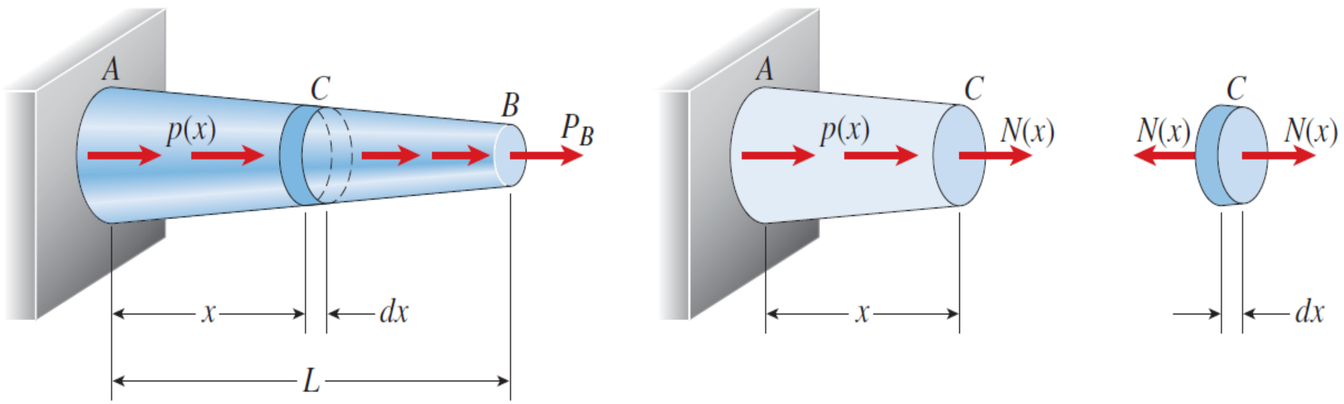
- The one-half is due to the assumption that external loading is applied gradually, starting from zero.

Strain Energy

- Nonuniform tension/compression



$$U=\sum_i U_i=\sum_i \frac{F_{Ni}^{2}L_i}{2E_iA_i}$$

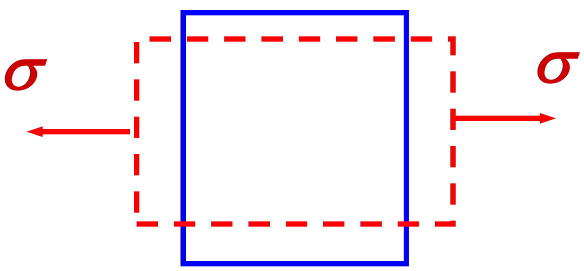


$$U=\int_0^L dU=\int_0^L \frac{F_N^{2}(x)dx}{2EA(x)}$$

Strain Energy Density (Energy per Unit Volume)

- Consider a differential cube of side length *a*, *b* and *c*

$$U = \frac{1}{2} F \Delta L = \frac{1}{2}\left[\sigma(bc)\right](a\varepsilon)$$



$$\Rightarrow u = \frac{U}{V} = \frac{1}{2}\sigma\varepsilon$$

- Total strain energy calculated from density

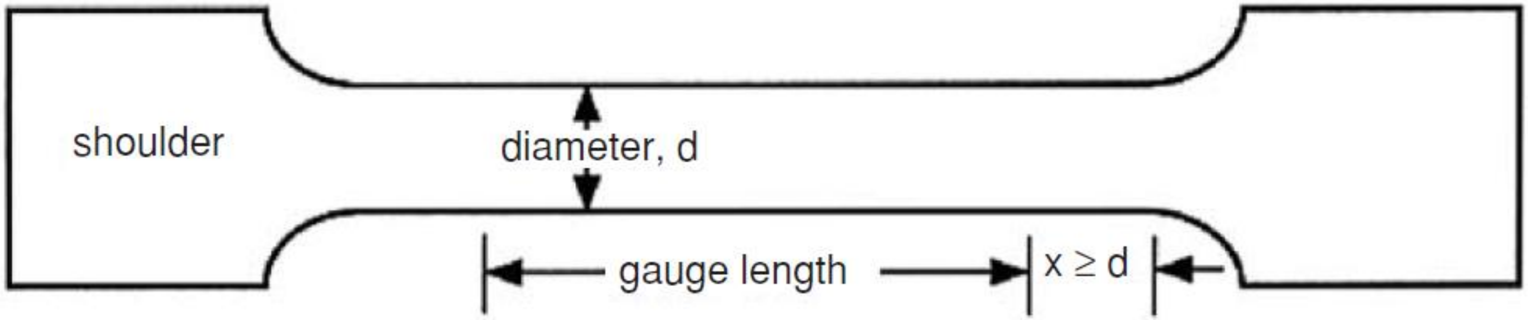
$$U = \int_V u dV = \int_V \frac{1}{2}\sigma\varepsilon dV = A\int_L \frac{1}{2}\sigma\varepsilon dx = A\int_L \frac{1}{2}\frac{F_\mathrm{N}}{A}\frac{F_\mathrm{N}}{EA}dx = \int_L \frac{F_\mathrm{N}^2 dx}{2EA}$$

- For constant E, F_N, and A

$$U = \frac{F_N^2 L}{2EA}$$

Mechanical Behavior of Materials

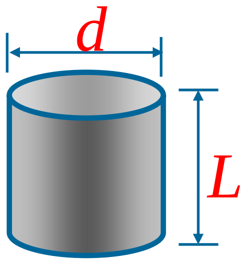
- Mechanical behavior of materials focus on the strength and deformation characteristics of solid materials under external loading.
- Tensile specimen

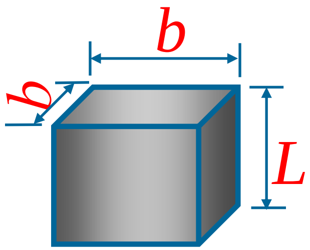


Typical tensile specimen with a reduced gauge section and larger shoulders.

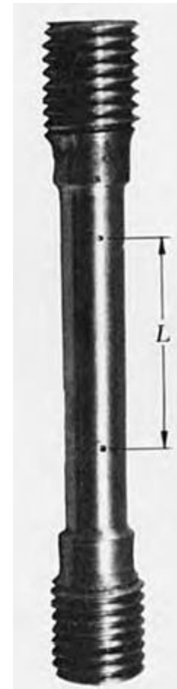
- For circular tensile specimen
  $L = 10d\ , 5d = 11.3\sqrt{A}, 5.65\sqrt{A}$
- For square tensile specimen
  $L = 11.3b, 5.65b.$

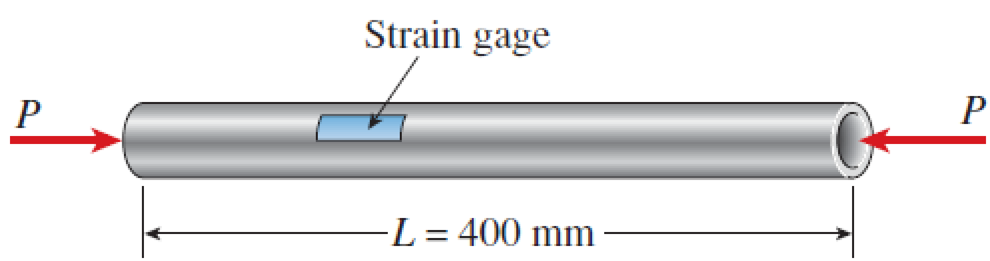
- Compressive specimen: $L = 1\sim3d, 1\sim3b$



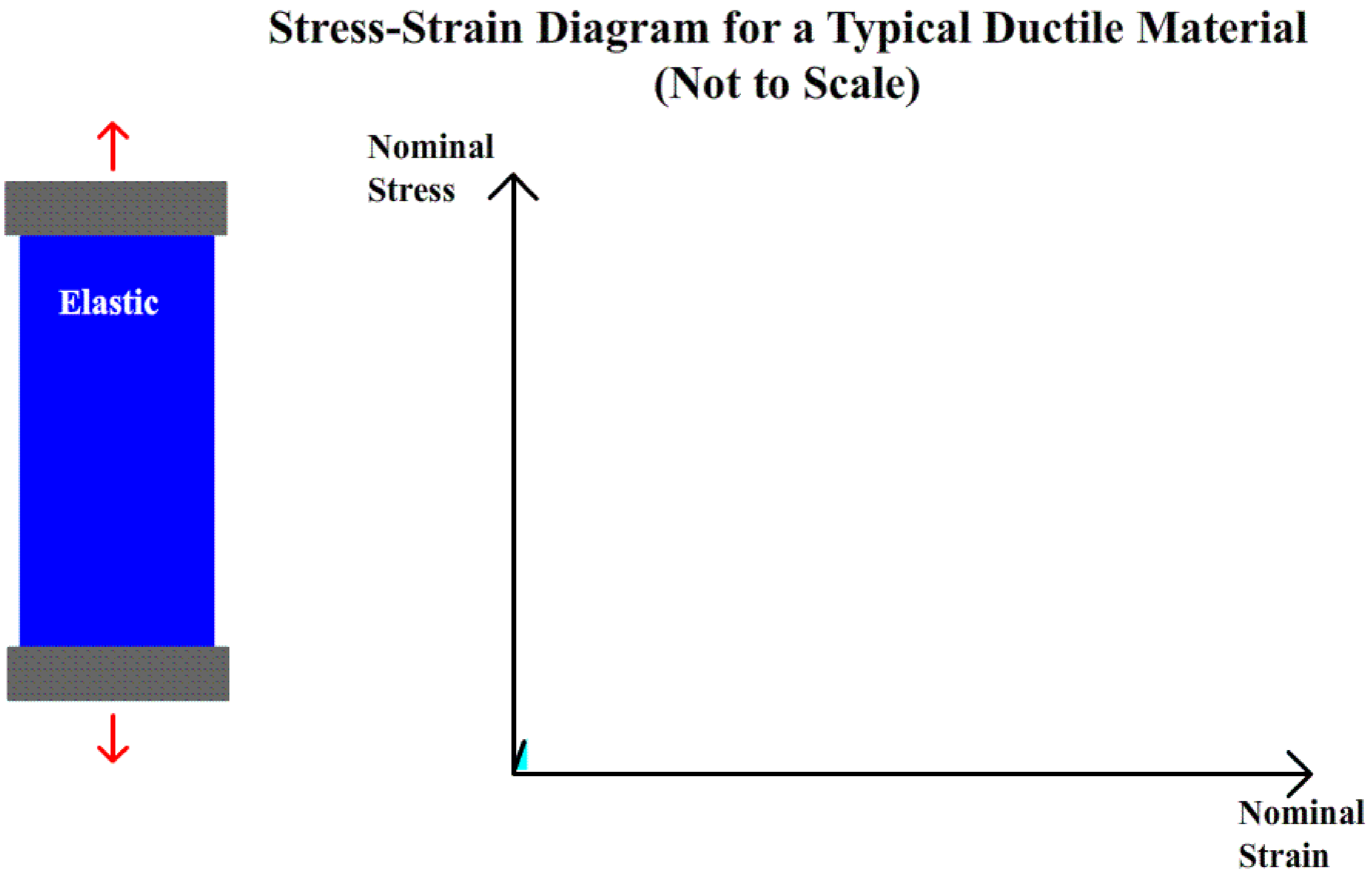


Mechanical Behavior of Materials



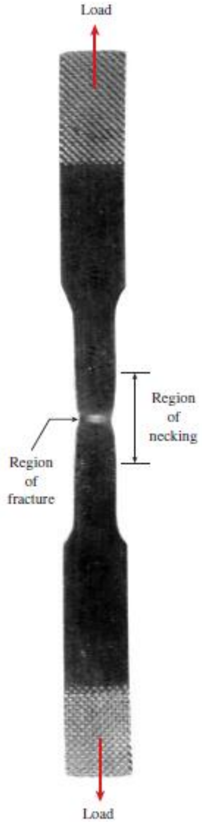


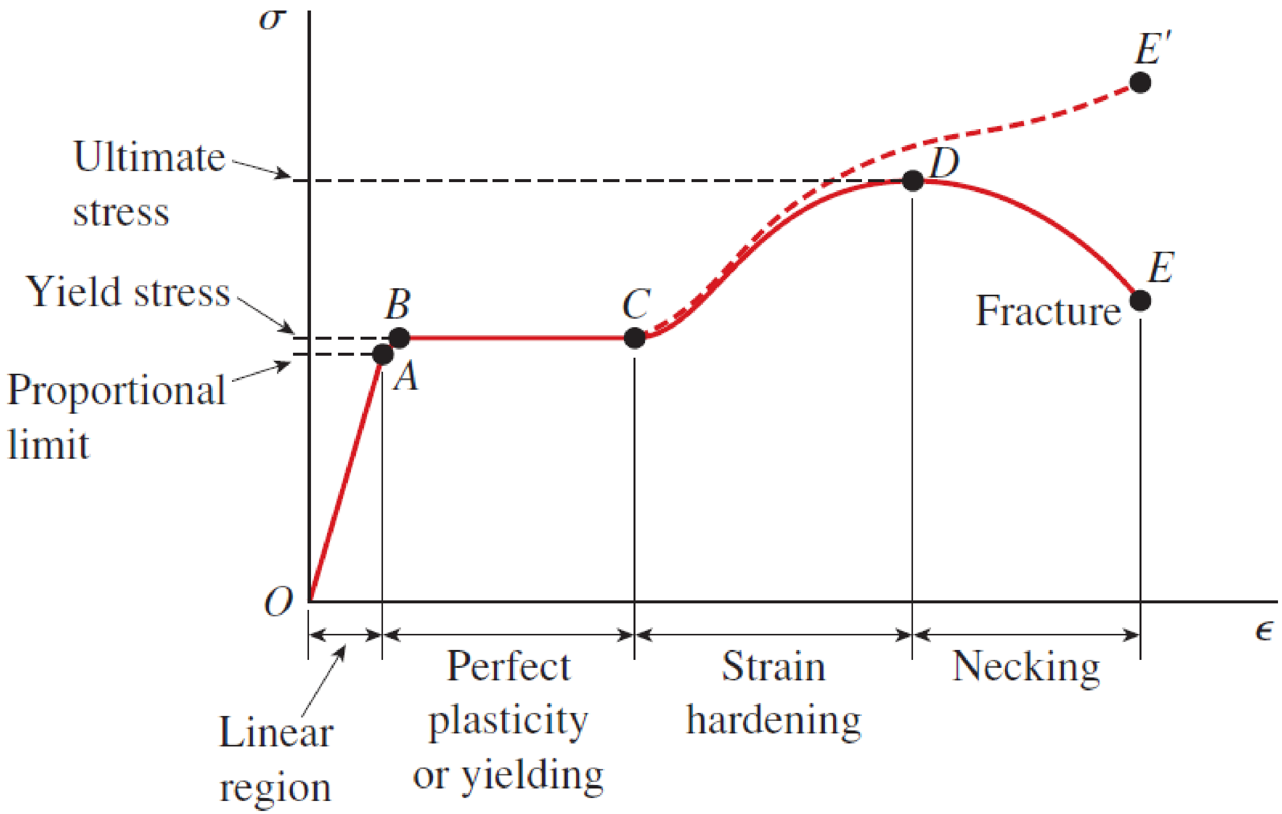
Nominal Stress-strain Curve



- Find more animations at http://em2lab.yolasite.com/.

Stress and Deformation Indices of Low-carbon Steel





- σ_p: Proportional limit
- σ_e: Elastic limit
- σ_Y: Yield stress
- σ_u: Ultimate stress

- Percent elongation:

  $\delta = (L_{\text{final}} - L_{\text{initial}}) / L_{\text{initial}} \times 100\%$

- Percent reduction in area

  $\psi = (A_{\text{initial}} - A_{\text{final}}) / A_{\text{initial}} \times 100\%$

Yield Stress, Ultimate Stress and Percent Elongation

Material	Yield stress σ_Y		Ultimate stress σ_U		Percent elongation (2 in. gage length)
	ksi	MPa	ksi	MPa	
Aluminum alloys	5–70	35–500	15–80	100–550	1–45
2014-T6	60	410	70	480	13
6061-T6	40	270	45	310	17
7075-T6	70	480	80	550	11
Brass	10–80	70–550	30–90	200–620	4–60
Bronze	12–100	82–690	30–120	200–830	5–60
Cast iron (tension)	17–42	120–290	10–70	69–480	0–1
Cast iron (compression)			50–200	340–1,400	
Concrete (compression)			1.5–10	10–70	
Copper and copper alloys	8–110	55–760	33–120	230–830	4–50
Glass			5–150	30–1,000	0
Plate glass			10	70	
Glass fibers			1,000–3,000	7,000–20,000	
Magnesium alloys	12–40	80–280	20–50	140–340	2–20
Monel (67% Ni, 30% Cu)	25–160	170–1,100	65–170	450–1,200	2–50
Nickel	15–90	100–620	45–110	310–760	2–50

Yield Stress, Ultimate Stress and Percent Elongation

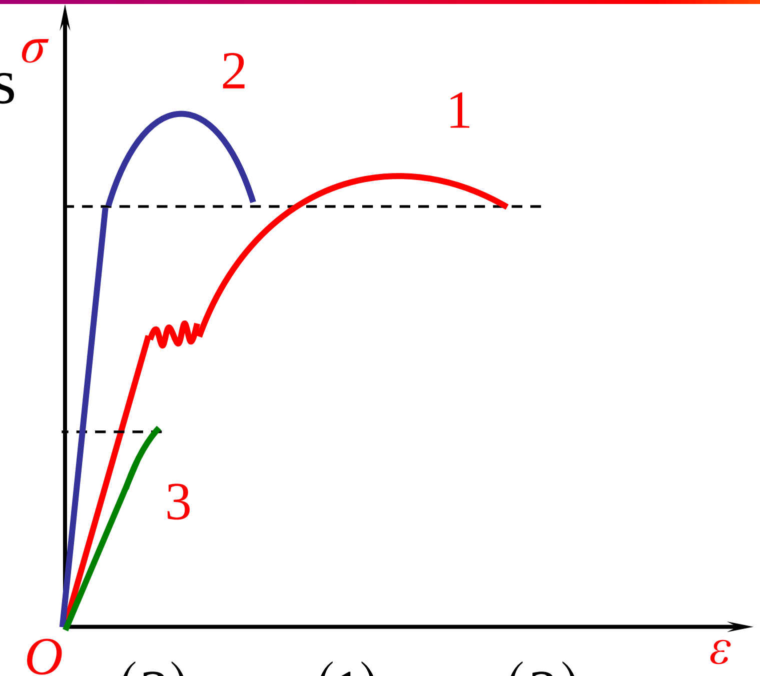
Material	Yield stress σ_Y		Ultimate stress σ_U		Percent elongation (2 in. gage length)
	ksi	MPa	ksi	MPa	
Steel wire	40–150	280–1,000	80–200	550–1,400	5–40
Titanium alloys	110–150	760–1,000	130–170	900–1,200	10
Tungsten			200–600	1,400–4,000	0–4
Wood (bending)					
Douglas fir	5–8	30–50	8–12	50–80	
Oak	6–9	40–60	8–14	50–100	
Southern pine	6–9	40–60	8–14	50–100	
Wood (compression parallel to grain)					
Douglas fir	4–8	30–50	6–10	40–70	
Oak	4–6	30–40	5–8	30–50	
Southern pine	4–8	30–50	6–10	40–70	

Yield Stress, Ultimate Stress and Percent Elongation

Material	Yield stress σ_Y		Ultimate stress σ_U		Percent elongation (2 in. gage length)
	ksi	MPa	ksi	MPa	
Plastics					
Nylon			6–12	40–80	20–100
Polyethylene			1–4	7–28	15–300
Rock (compression)					
Granite, marble, quartz			8–40	50–280	
Limestone, sandstone			3–30	20–200	
Rubber	0.2–1.0	1–7	1–3	7–20	100–800
Steel					
High-strength	50–150	340–1,000	80–180	550–1,200	5–25
Machine	50–100	340–700	80–125	550–860	5–25
Spring	60–240	400–1,600	100–270	700–1,900	3–15
Stainless	40–100	280–700	60–150	400–1,000	5–40
Tool	75	520	130	900	8
Steel, structural	30–100	200–700	50–120	340–830	10–40
ASTM-A36	36	250	60	400	30
ASTM-A572	50	340	70	500	20
ASTM-A514	100	700	120	830	15

Exercise

- Based on the three stress-strain curves shown in the figure, which one of the following regarding ultimate stress, Young's modulus and percent elongation is correct?



A:
$$\sigma_u(1) = \sigma_u(2) > \sigma_u(3)$$
$$E(1) > E(2) > E(3)$$
$$\varepsilon(1) > \varepsilon(2) > \varepsilon(3)$$

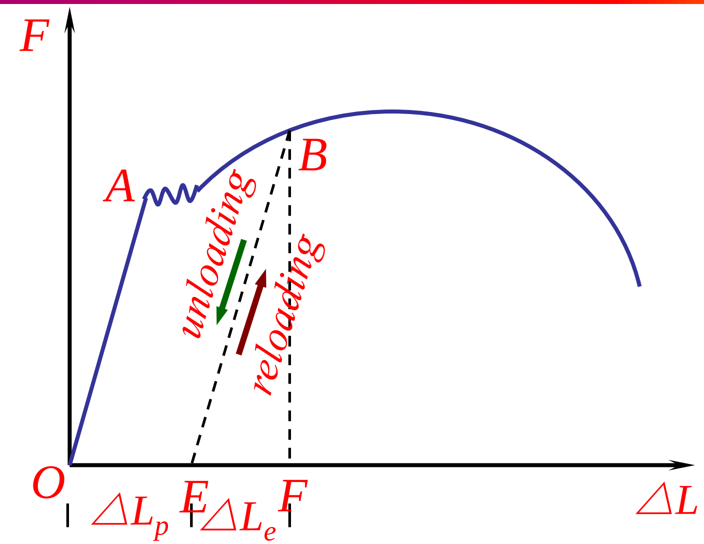
B:
$$\sigma_u(2) > \sigma_u(1) > \sigma_u(3)$$
$$E(2) > E(1) > E(3)$$
$$\varepsilon(1) > \varepsilon(2) > \varepsilon(3)$$

C:
$$\sigma_u(3) < \sigma_u(1) < \sigma_u(2)$$
$$E(3) > E(1) > E(2)$$
$$\varepsilon(3) > \varepsilon(2) > \varepsilon(1)$$

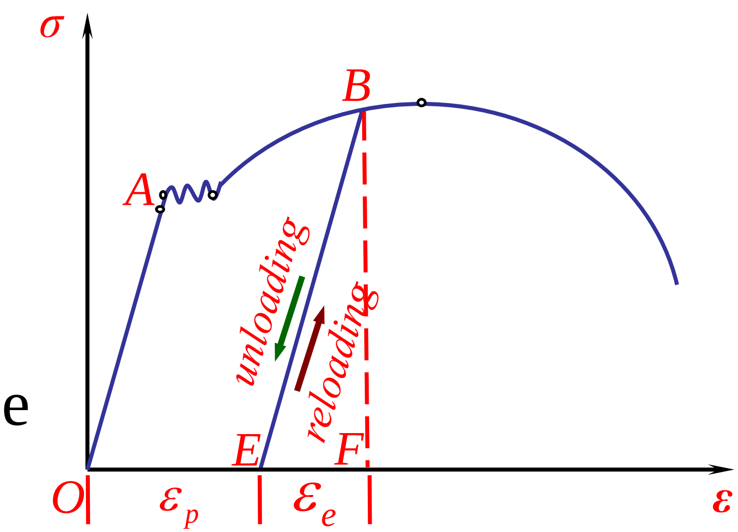
D:
$$\sigma_u(1) > \sigma_u(2) > \sigma_u(3)$$
$$E(2) > E(1) > E(3)$$
$$\varepsilon(2) > \varepsilon(1) > \varepsilon(3)$$

Strain (Work) Hardening

- Unloading at point *B* during hardening stage down to zero force. *EB* almost parallel to *OA*. (Unloading law)
- Deformation in the hardening stage is composed of elastic ($\triangle L_e$) and plastic elongation ($\triangle L_p$).
- If the test sample is reloaded at point *E*, the linear elastic (proportional) limit substantially increases. As a result, the overall deformation the sample can endure decreases.
- In engineering practice, strain hardening is often employed to increase the maximum resistance force within linear elastic scope.



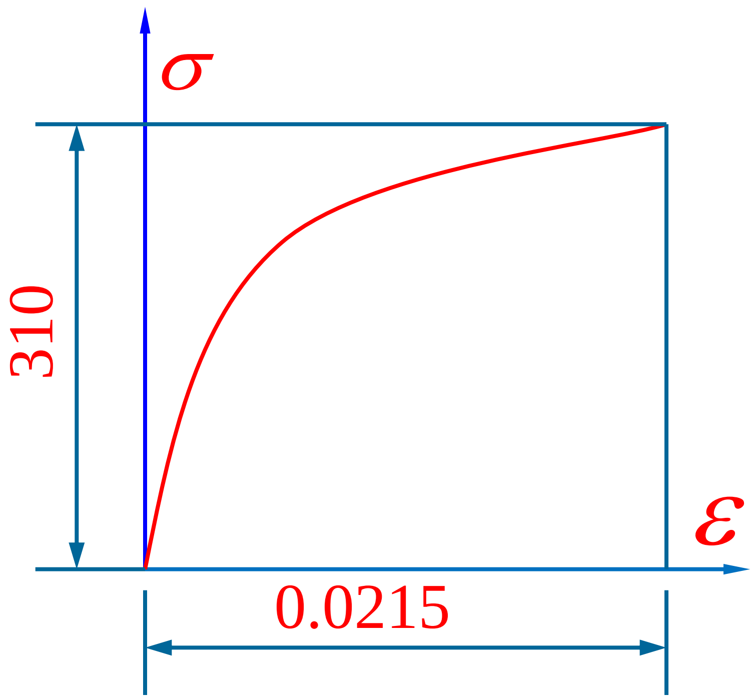
Force-displacement Diagram



Stress-strain Diagram

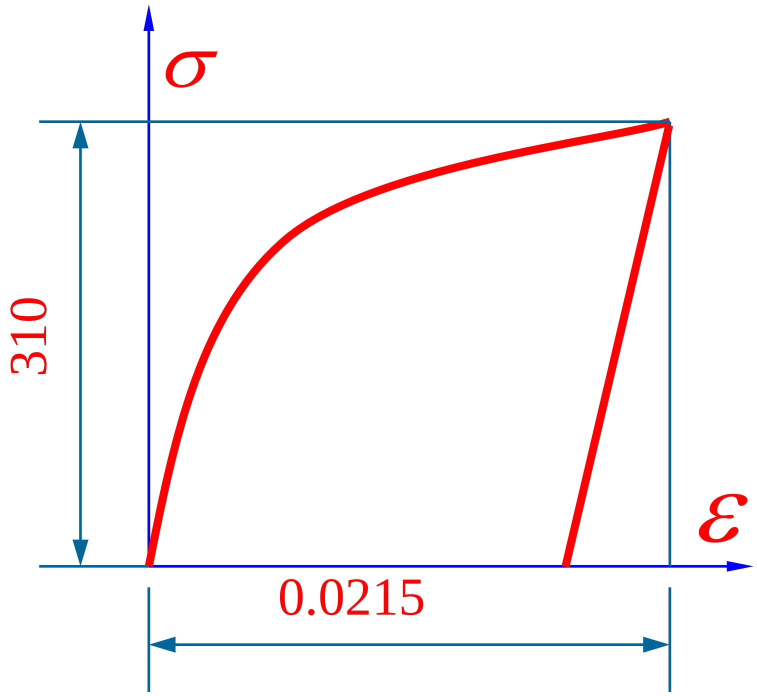
Sample Problem

- Given: E = 200 GPa, unloading at σ = 310 MPa during hardening stage, total strain $\varepsilon_e + \varepsilon_p$ = 0.02155. Find: ε_e and ε_p.



- Solution:

$$\varepsilon_e = \frac{\sigma}{E} = \frac{310}{200 \times 1000} = 0.00155$$



$$\varepsilon_p = \varepsilon - \varepsilon_e = 0.02 >> \varepsilon_e$$

- Remark: in experimental reports, strain often takes the unit of micro-strain, i.e. 0.02 (ε)=20000 (με)

$$1\mu\varepsilon = 10^{-6}\varepsilon$$

Sample Problem

- Low-carbon steel loading and unloading experiments. Given: E = 210 GPa, σ_P = 210 MPa. Find: 1. normal stress at the strain level ε = 0.001 (under linear elastic limit); 2. normal stress at a point in hardening stage, unloading from which results in ε = 0.08 and ε_p = 0.078.

- Solution：

1. Under linear elastic limit

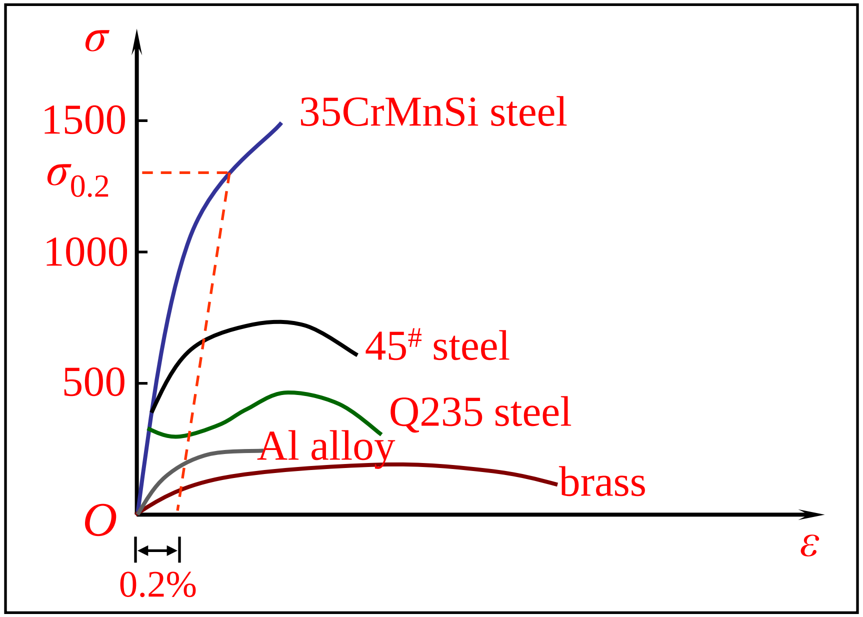
$$\sigma = E\varepsilon = 210\times10^9\times0.001(\text{Pa}) = 210(\text{MPa}) \le \sigma_p$$

2. During hardening stage:

$$\sigma = E\varepsilon_e = 210\times10^9\times(0.08-0.078)(\text{Pa}) = 420(\text{MPa})$$

Mechanical Behavior of General Ductile Materials under Tension

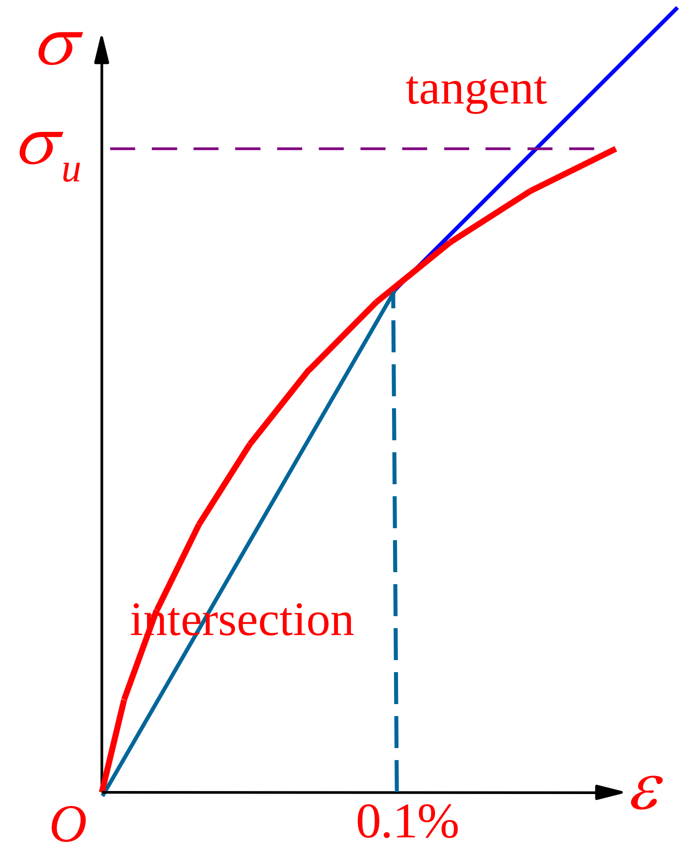
- Possess distinctive σ_Y and σ_u
- May not have yielding and/or localized deformation stage
- Relatively large extension rate after fracture ($\delta \geq 5\%$)



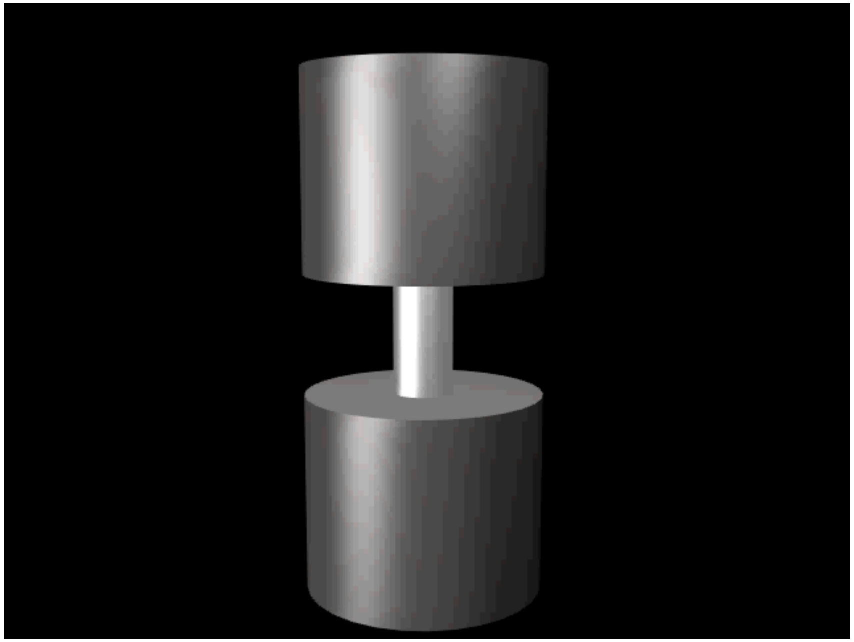
- If there is no obvious yielding stage: take the normal stress corresponding to 0.2% plastic strain as yield limit ($\sigma_{p0.2}$)

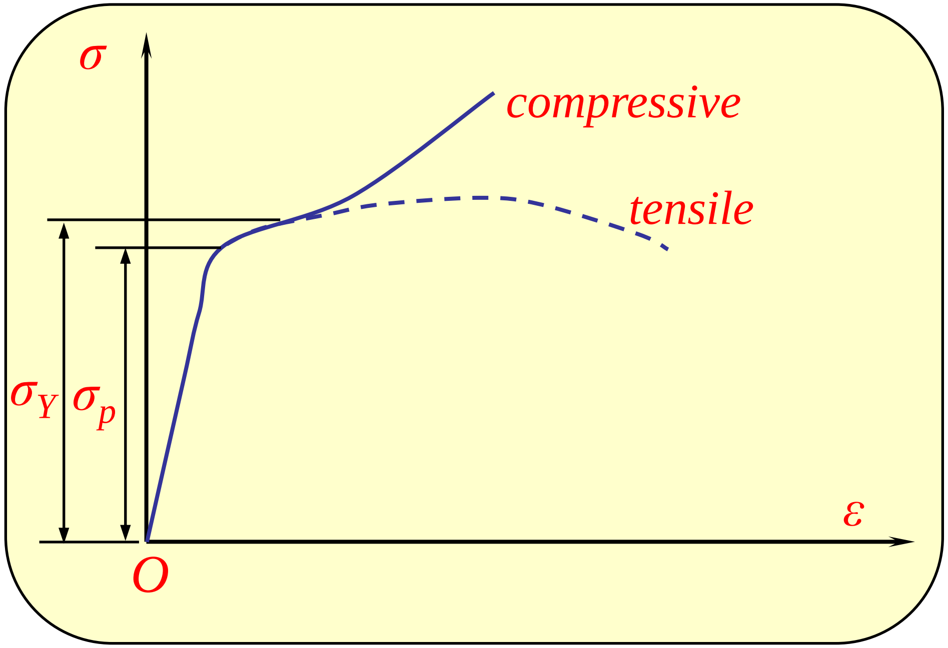
Mechanical Behavior of Brittle Materials under Tension

- σ-ε is a slightly curved line and approximately obeys the Hooke's law
- No yielding, hardening and localized deformation stage
- The ultimate stress is the only index
- Relatively small percent elongation after fracture (δ = 2%-5%)
- Tangential modulus: slope at any point of σ-ε curve
- secant modulus: can be defined at ε = 0.1%



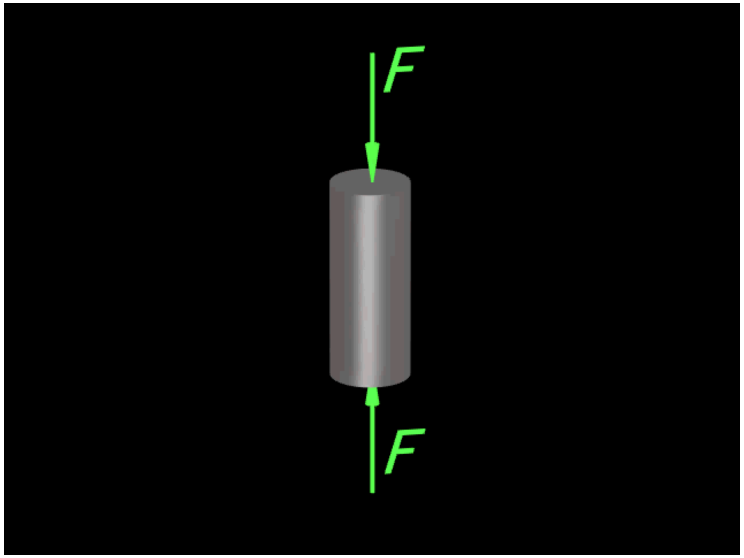
Mechanical Behavior of Low-carbon Steel under Compression

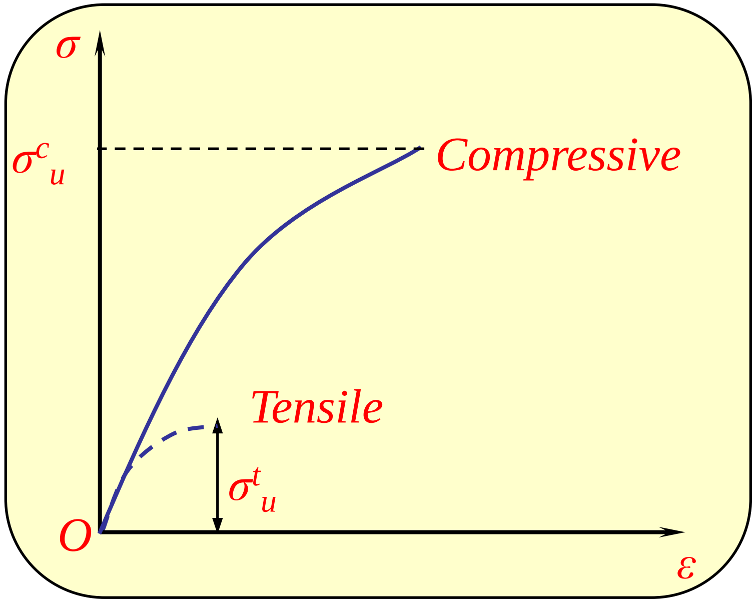




- E, σ_p, σ_Y take the same values as in tensile tests.
- Cannot obtain compressive ultimate stress due to the (constant) initial cross-sectional area used in normal stress calculation.
- Compressive mechanical behavior of low-carbon steel is obtained from its tensile indices.

Mechanical Behavior of Cast Iron under Compression

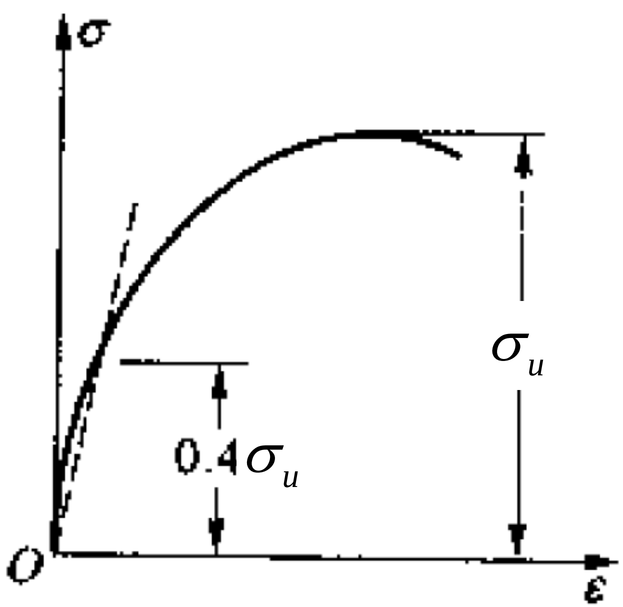


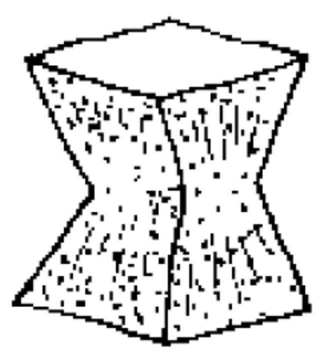


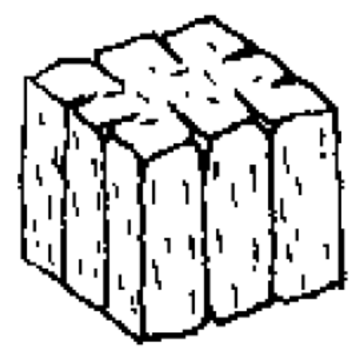
- There is only very short linear σ-ε curve (only approximately follows Hooke's law).
- Missing yielding stage (no obvious σ_Y).
- Much larger compressive ultimate stress (typically 4-5 times that of tensile strength limit).
- Final damage is in the form of shearing along a surface 45^0 from axis.

Mechanical Behavior of Concrete

- Compressive ultimate stress is much larger than its tensile counterpart (5-20 times)
- Compressive behavior depends on the frictional state at sample ends
- Under frictional end-condition, sample is composed of two trapezoidal cones under mirror-symmetry upon damage
- σ-ε comprises a short straight line, followed by an obvious curve.
- Elastic modulus can be defined by the slope of secant line at $\sigma = 0.4\sigma_u$
- Small compressive ratio after fracture.

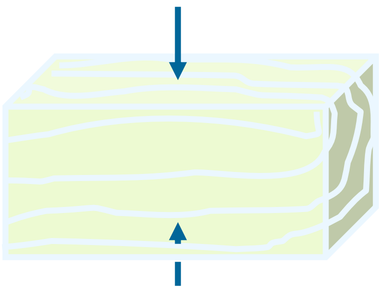




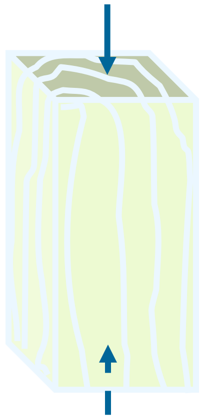


Mechanical Behavior of Wood

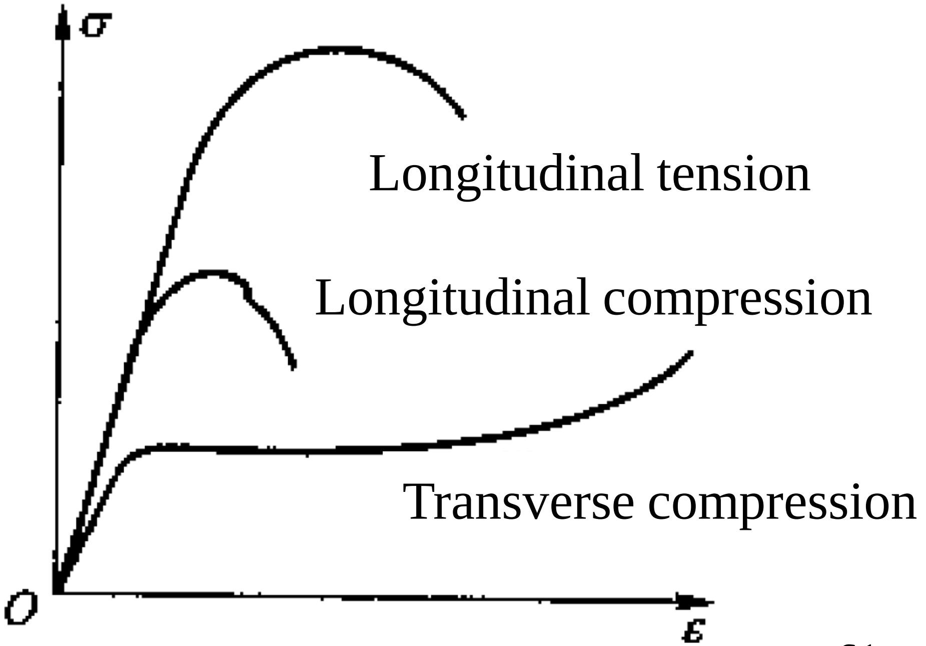
- Material behavior is transversely isotropic.
- Longitudinal tensile strength is unstable due to knots.
- Longitudinal compressive strength is insensitive to knots.
- Much larger longitudinal compressive strength than the transverse one.
- Most often used as compressive or support bars in engineering



Transverse compression



Longitudinal Compression



Mechanical Behavior of Composite Materials

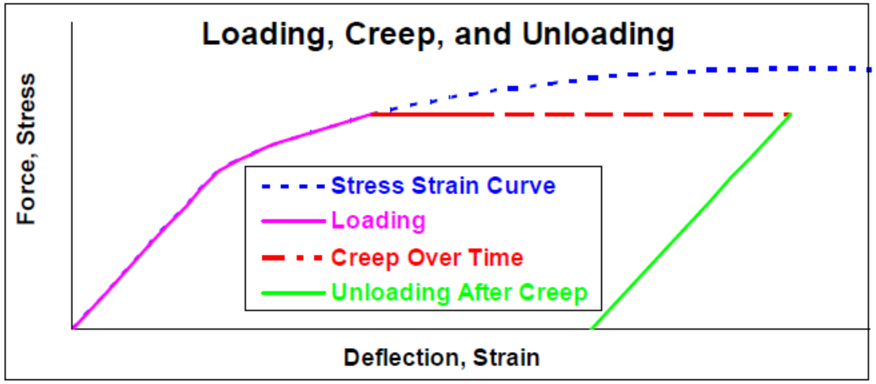
- Mechanical properties of composite materials is mostly determined by the way how fibers are arranged (anisotropic).

Influence from temperature, stress concentration (geometric factors), and strain rate (visco-elasticity)

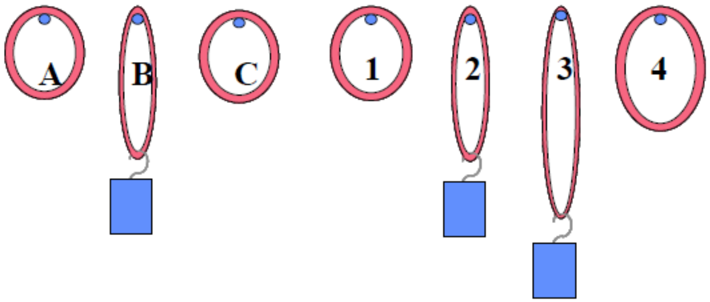
- High ultimate stress and low plasticity under low temperature.
- Low ultimate stress and low plasticity under high stress concentration state
- Low ultimate stress and high plasticity under high strain rate

Mechanical Behavior of Viscoelastic Materials

- $\sigma = \varepsilon \cdot f(t)$ - linear visco-elasticity
- $\sigma = f(\varepsilon, t)$ - non-linear visco-elasticity
- Creep: increasing strain with time under constant stress

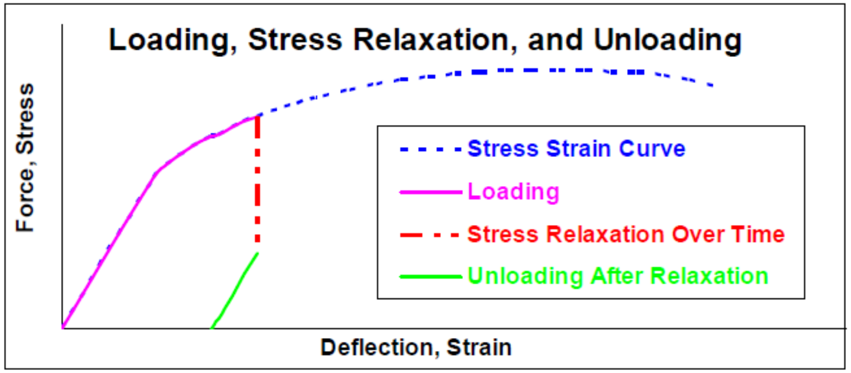


Stress-Strain Curve with Creep

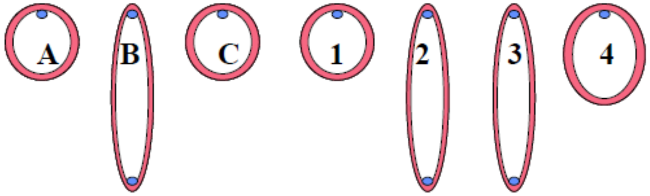


Creep Example

- Relaxation: decreasing stress with time under constant strain



Stress-Strain Curve with Stress Relaxation



. Stress Relaxation Example

Strength Condition

- **Limit stress (σ_{lim}):** the stress under which mechanical components get damaged.
- **Damage Criteria:** yielding (σ_Y) for ductile materials; fracture (σ_u) for brittle materials
- **Allowable Stress [σ]:** the maximum stress allowed in engineering practice. It is typically taken as one n^{th} (Safety Factor) of the limit stress
- **Strength Condition:**

$$\sigma_{max} \le \frac{\sigma_{lim}}{n} = [\sigma]$$

- **Strength Analysis:**

(1) Strength check: $\sigma_{max} \le [\sigma]$

(2) Cross-section design: $A \ge F_{N\max}/[\sigma]$

(3) Find allowable load: $F_{N\max} \le A[\sigma]$

Sample Problem

- Given: d = 14 mm, [σ] = 170 MPa, uniaxial tension load F = 2.5 kN. Check the strength condition of the circular bar.

- Solution

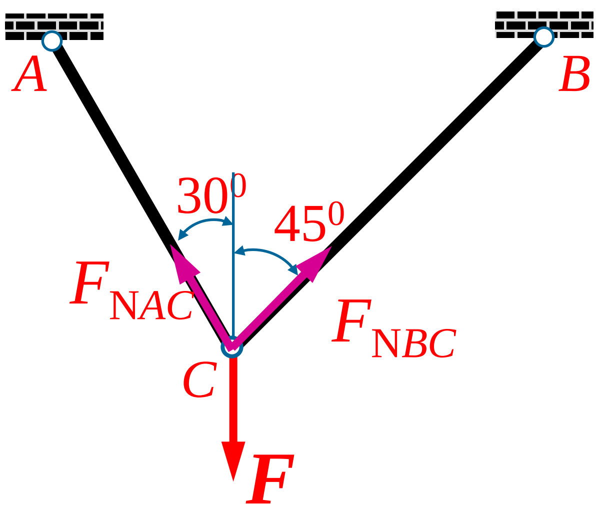
$$\sigma_{\max} = \frac{F_{\mathrm{N,max}}}{A} = \frac{2.5\times10^{3}}{\frac{\pi}{4}\times14^{2}\times10^{-6}}\,\mathrm{Pa} = 162\ \mathrm{MPa} < [\sigma]$$

- The strength condition is satisfied.

Sample Problem

- Given: $A_{AC} = 450\ \text{mm}^2$, $A_{BC} = 250\ \text{mm}^2$, $E_{AC} = E_{BC}$, $[\sigma] = 100$ MPa.
  Find: the maximum allowable load F of the truss.

- Solution:



1. Find F_{NAC} and F_{NBC}

Method of joint at C:

$$\sum F_x = 0,\ F_{NBC}\sin 45^0 - F_{NAC}\sin 30^0 = 0$$

$$\sum F_y = 0,\ F_{NBC}\cos 45^0 - F_{NAC}\cos 30^0 - F = 0$$

We get: $F_{NAC} = 0.732F$, $\quad F_{NBC} = 0.517F$

2. Allowable axial force in bar AC and BC

$$F_{\mathrm{N}AC} = 0.732F \le A[\sigma] = 450\times10^{-6}\,\mathbf{m}^2 \times 100\times10^{6}\,\mathbf{Pa}$$

so: $[F] \le 61.48$ kN

$$F_{\mathrm{N}BC} = 0.517F \le A[\sigma] = 250\times10^{-6}\,\mathbf{m}^2 \times 100\times10^{6}\,\mathbf{Pa}$$

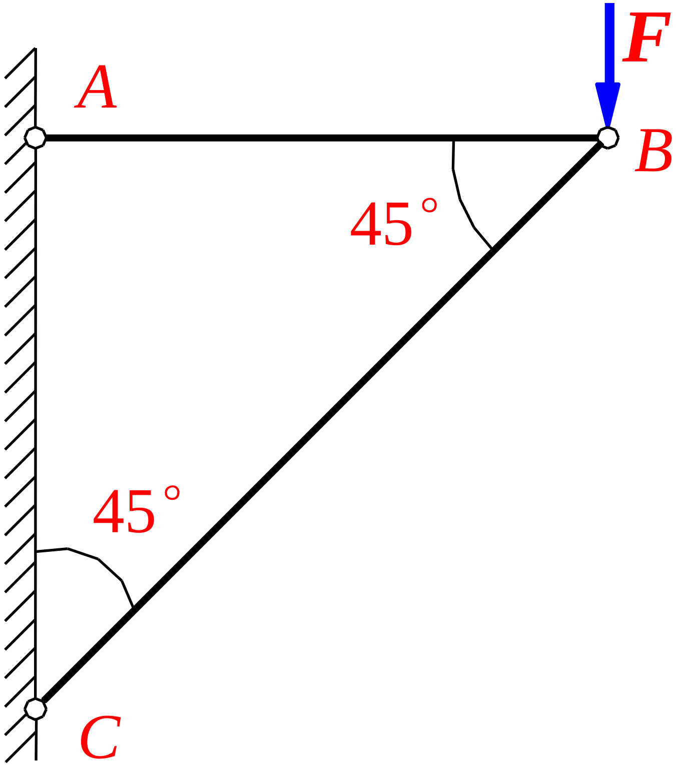
So: $[F] \le 48.36$ kN

3. Allowable load $[F]$

Take the smaller value: $[F] \le 48.36$ kN.

Sample Problem

- Given: F = 75 kN, $[\sigma]$ = 160 MPa. Find: the minimum A_{AB} and A_{BC}.
- Solution:



1. Find F_{NAB} and F_{NBC}

By the Method of joint at B:

$$\sum F_y = 0,\ F_{NBC}\cos 45^0 + F = 0$$

$$\sum F_x = 0,\ F_{NBC}\cos 45^0 + F_{NAC} = 0$$

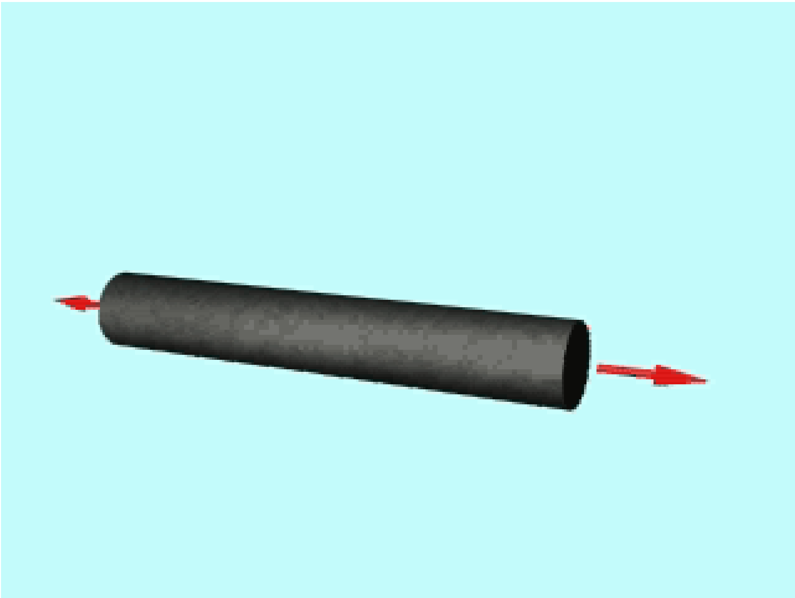
We get: $F_{NAC} = F$, $F_{NBC} = -1.414F$

2. Minimum cross-sectional area

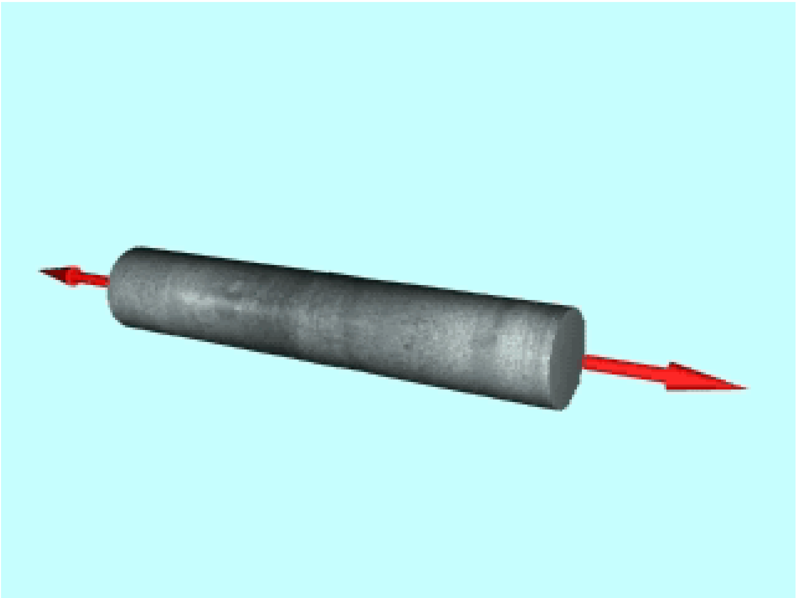
$$A_{AB} \geq \frac{F_{NAB}}{[\sigma]} = \frac{75\times 10^3}{160\times 10^6} = 4.687\times 10^{-4}\ \text{m}^2 = 4.687\ \text{cm}^2$$

$$A_{BC} \geq \frac{|F_{NBC}|}{[\sigma]} = \frac{106\times 10^3}{160\times 10^6} = 6.629\times 10^{-4}\ \text{m}^2 = 6.629\ \text{cm}^2$$

Failure of Brittle vs. Ductile Bars under Tension



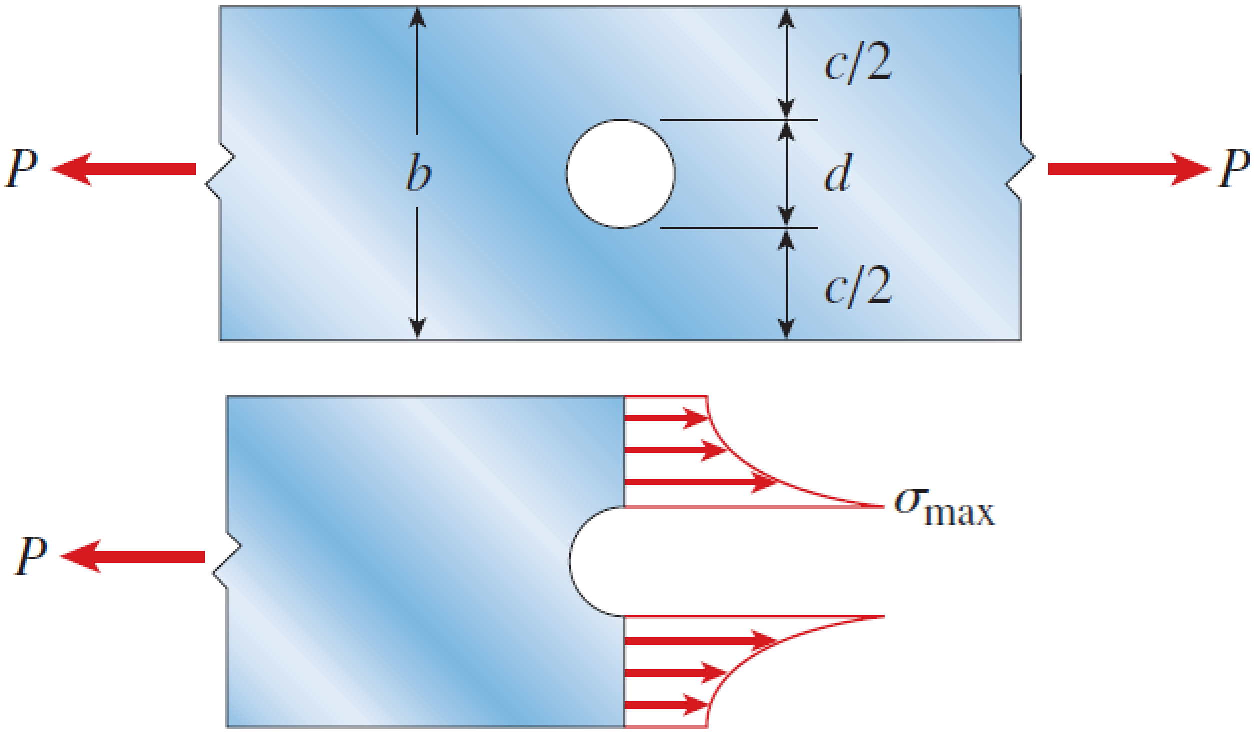
Brittle



Ductile

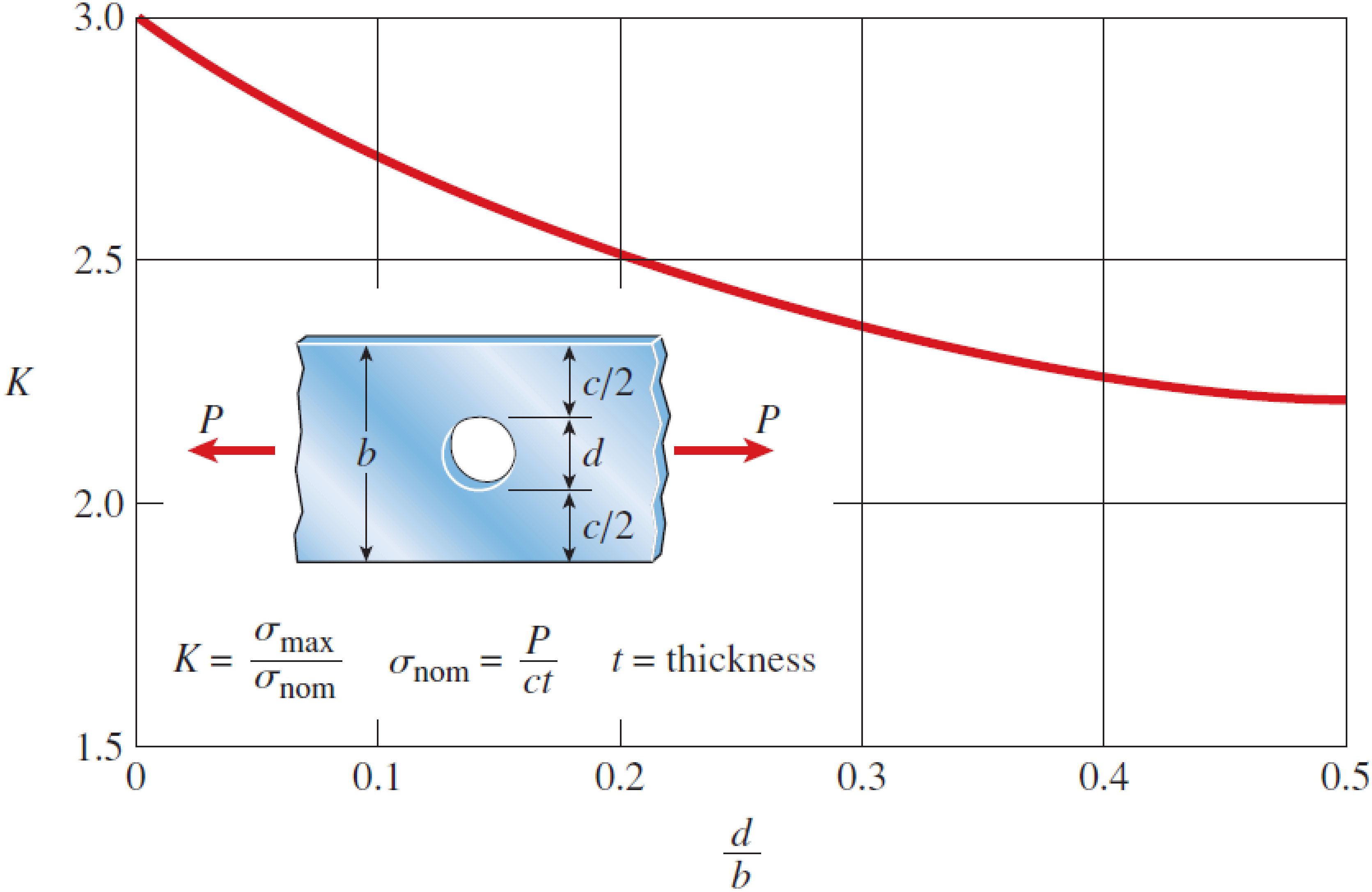
- Under uniaxial tension, brittle bars break along cross-sections while ductile ones glide along 45^0 sections during yielding?

Stress Concentration

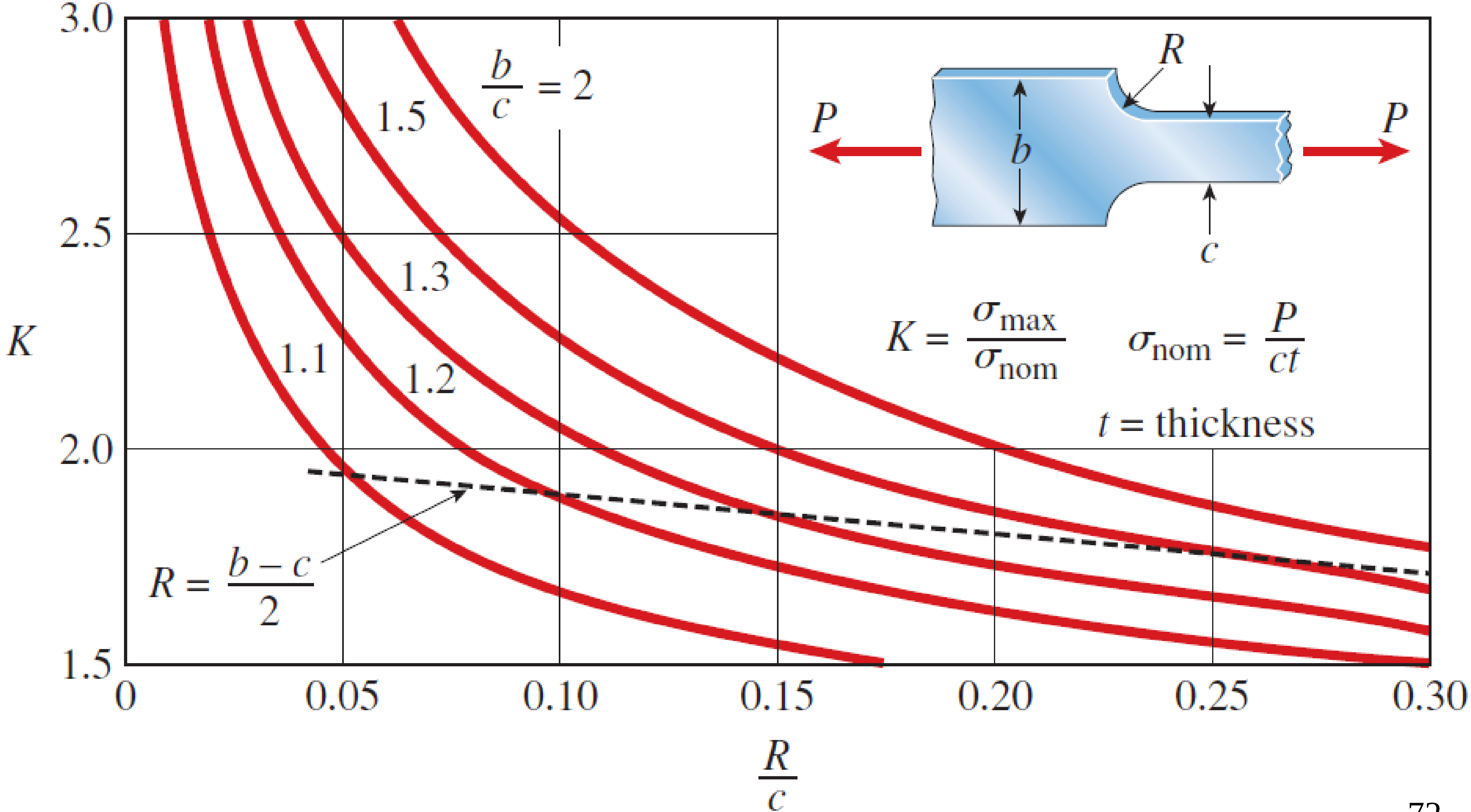


- **Stress concentration:** rapid stress increase at specific locations where geometric defects and/or abrupt cross-sectional area change occurs

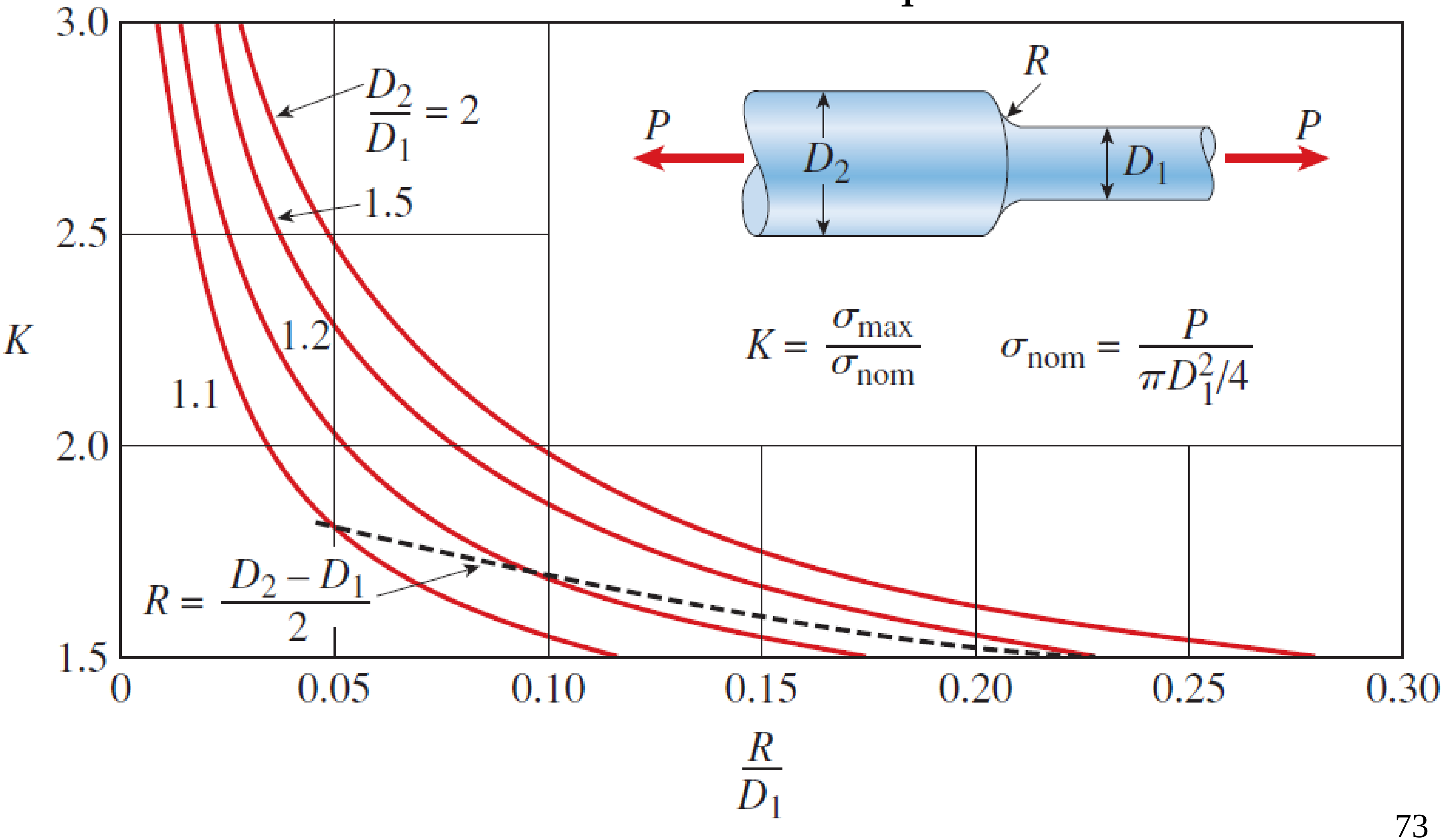
- Stress-concentration factor for flat bars with circular holes



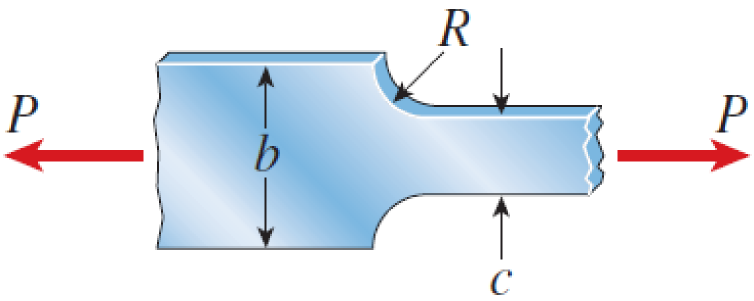
- Stress-concentration factor for flat bars with shoulder fillets. The dashed line is for a full quarter-circular fillet.



- Stress-concentration factor for round bars with shoulder fillets. The dashed line is for a full quarter-circular fillet.



Sample Problem



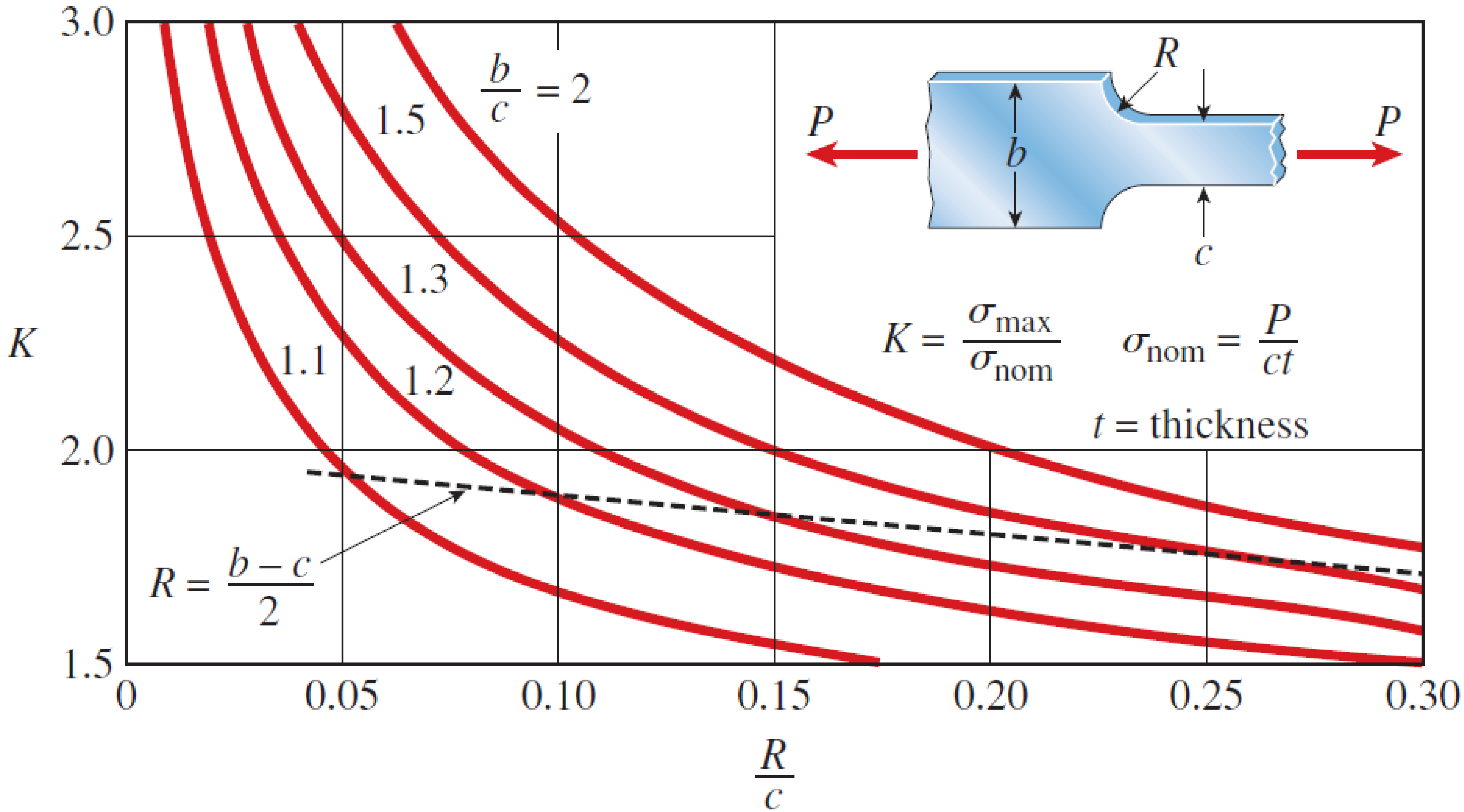
Determine the largest axial load P that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively c = 40 and b = 60 mm wide, connected by fillets of radius R = 8 mm. Assume an allowable normal stress of 165 MPa.

SOLUTION:

- Determine the geometric ratios and the stress concentration factor.
- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.
- Apply the definition of normal stress to find the allowable load.

- Determine the geometric ratios and the stress concentration factor.

$$\frac{b}{c} = \frac{60\,\text{mm}}{40\,\text{mm}} = 1.50 \qquad \frac{R}{c} = \frac{8\,\text{mm}}{40\,\text{mm}} = 0.20 \qquad \Rightarrow K = 1.82$$



- Determine the geometric ratios and the stress concentration factor.

$$\frac{b}{c} = \frac{60\,\text{mm}}{40\,\text{mm}} = 1.50 \qquad \frac{R}{c} = \frac{8\,\text{mm}}{40\,\text{mm}} = 0.20 \qquad \Rightarrow K = 1.82$$

- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.

$$\sigma_{\text{ave}} = \frac{\sigma_{\text{max}}}{K} = \frac{165\,\text{MPa}}{1.82} = 90.7\,\text{MPa}$$

- Apply the definition of normal stress to find the allowable load.

$$P = A\sigma_{ave} = (40\,\text{mm})(10\,\text{mm})(90.7\,\text{MPa}) = 36.3\times10^{3}\,\text{N}$$

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